The Most-Cited Physical-Sciences Publications in the 1945-1954 *Science Citation Index*. Part 2. Mathematics

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This essay examines 20 highly cited papers in mathematics, based on the *Science Citation Index*® cumulation for 1945-1954. Next week 42 most-cited papers in astronomy and the earth sciences will be examined. These papers are compared with other publications (including some highly cited books) considered important by scientists and historians of science. The essay discusses some of the major trends, achievements, and researchers in mathematics in the period including World War II.

Introduction: Finding Highly Cited Publications in Small Fields

In Part 1 of this essay, I discussed 52 highly cited publications in the physical sciences, based on the *Science Citation Index*® (*SCI*®) cumulation for 1945-1954.[1] That list was composed almost entirely of publications in chemistry (25) and physics (25); there were only two in mathematics, and none in astronomy or the earth sciences. Just as one cannot ignore the physical sciences merely because they generate fewer citations than the biological sciences,[2] one cannot simply ignore astronomy, the earth sciences, and mathematics merely because they generate fewer citations than physics and chemistry. ISI® has therefore generated additional lists of relatively highly cited papers in these smaller, less-cited fields. In addition, I present lists of publications considered important by scientists or historians of science.

Do Citations Measure Importance? The Case of Mathematics

As noted in Part 1 of this essay, one should not simply rely on citation counts as a measure of the importance or quality of a publication. Rather, it is desirable also to obtain the independent judgments of the scientific community—for example, as indicated by Nobel Prizes—or of historians of science. Thus, 48 percent of the most-cited physics publications and 40 percent of the most-cited chemistry publications were authored or coauthored by a Nobel laureate, although those publications were not necessarily the work for which they received the Nobel Prize.[1]

For mathematics, the closest equivalent to the Nobel Prize is the Fields Medal, awarded at the quadrennial International Congress of Mathematicians, beginning in 1936. No medals were given between 1936 and 1950; the medals awarded in 1936,

Table 1: Winners of the Fields Medal in mathematics, awarded at the International Congress of Mathematicians in 1936, 1950, and 1954, and their areas of research. Medalists are listed in alphabetic order. Dates in parentheses in the "Research Area" column give the time period when the medal-winning work was done.

Medalist	Year Awarded	Research Area
Ahlfors L	1936	Complex-variable theory, quasiconformal mappings, Riemann surfaces, meromorphic functions (1920s, 1930s).
Douglas J	1936	Solved Plateau problem (minimal surface) (1931).
Kodaira K	1954	Harmonic integrals and harmonic forms with application to Kahlerian and algebraic varieties (1944-1953).
Schwartz L	1950	Theory of distributions (1945-1951).
Selberg A	1950	Prime number theorem (1948-1949), Riemann zeta function (1940s).
Serre J-P	1954	Complex variables, cohomology in a complex-analytic sheaf (1950-1951).

1950, and 1954 were for research by six mathematicians published in the period from about 1930 to about 1952. These are listed in Table 1.

Table 2 presents 20 mathematics journal articles that were most cited in the 1945-1954 *SCI*. Comparing both tables, one can see that none of the Fields Medal winners appear as authors of the 20 most-cited mathematics articles during this period. The most-cited journal article by a Fields winner is by Jean-Pierre Serre, College of France, Paris.[3] Its 27 citations from 1945 to 1954, however, are too few to put it on the list of 20 most-cited mathematics papers, which were cited at least 30 times. Citations for the 1958 Fields Medal winners (Klaus F. Roth, University of London, UK, and René Thom, University of Strasbourg, France) were even fewer, so including them would not make any difference to our conclusion: the most important research in mathematics, as judged by awards of the Fields Medal, is not highly cited, and the most-cited publications in mathematics do not contain the most important research.

Some mathematicians would undoubtedly argue that Stefan Banach's (University of Lvov, USSR) *Théorie des opérations linéaires* is a counterexample to this generalization, since it showed up on the list of 52 most-cited physical-sciences papers and is generally regarded as a report of important original research.[1,4] Nevertheless, it did not win the 1936 Fields Medal for which it was presumably eligible.

The Most-Cited Mathematics Articles

The research areas of the Fields Medal winners and the most-cited papers published in mathematics journals indicate trends in pure mathematics during the 1930s and 1940s. Abstract algebra and topology were the most popular subjects. As Jean Dieudonné, University of Nice, France, expressed it in his survey of modern mathematics, the emphasis was on studying the structure rather than the content of mathematical objects.[5] Most of the highly cited mathematicians are listed as "originators" of one or more of the research specialties described by Dieudonné.[6]

The most-cited mathematics article is on statistics and was authored by Henry B. Mann and D.R. Whitney, Ohio State University, Columbus. Most of its 109 citations from 1945 to 1954 are from biological and medical journals, so one may question whether it should be included in a list of highly cited *physical-sciences* publications. Mann has described its origin in the problem of testing a drug that was supposed to protect against the common cold.[7]

One of the most-cited papers in mathematics journals was by Milton Friedman (b. 1912), then with the National Resources Committee, Washington, DC, who won the 1976 Nobel Prize for economics. Presumably, his 1937 paper on the use of rank ordering in statistical analysis was only a small part of the body of work for which he was honored, and the award of the Nobel Prize

Table 2: The 20 most-cited papers from mathematics journals covered in the 1945-1954 *SCI®* cumulation. Papers are listed in alphabetic order by first author. A=total number of 1945-1954 citations.

A	Bibliographic Data
30	**Bartlett M S.** On the theoretical specification and sampling properties of autocorrelated time-series. *J. Roy. Statist. Soc. Ser. B Metho.* 8:27-41, 1946.
31	**Berkson J.** Application of the logistic function to bio-assay. *J. Amer. Statist. Assn.* 39:357-65, 1944.
33	**Friedman M.** The use of ranks to avoid the assumption of normality implicit in the analysis of variance. *J. Amer. Statist. Assn.* 32:675-701, 1937.
32	**Iwasawa K.** On some types of topological groups. *Ann. Math.* 50:507-58, 1949.
61	**Jacobson N.** The radical and semi-simplicity for arbitrary rings. *Amer. J. Math.* 67:300-20, 1945.
38	**Jacobson N.** Structure theory of simple rings without finiteness assumptions. *Trans. Amer. Math. Soc.* 57:228-45, 1945.
30	**Kakutani S.** Concrete representation of abstract (*M*)-spaces (A characterization of the space of continuous functions). *Ann. Math.* 42:994-1024, 1941.
31	**King R & Middleton D.** The cylindrical antenna; current and impedance. *Quart. Appl. Math.* 3:302-35, 1946.
35	**Lin C C.** On the stability of two-dimensional parallel flows. Part I.—General theory. *Quart. Appl. Math.* 3:117-42, 1945.
109	**Mann H B & Whitney D R.** On a test of whether one of two random variables is stochastically larger than the other. *Ann. Math. Statist.* 18:50-60, 1947.
30	**Middleton D.** Some general results in the theory of noise through non-linear devices. *Quart. Appl. Math.* 5:445-98, 1947.
32	**Murnaghan F D.** Finite deformations of an elastic solid. *Amer. J. Math.* 59:235-60, 1937.
41	**Murray F J & von Neumann J.** On rings of operators. *Ann. Math.* 37:116-229, 1936.
33	**Neyman J.** On a class of "contagious" distributions, applicable in entomology and bacteriology. *Ann. Math. Statist.* 10:35-57, 1939.
32	**Steenrod N E.** Products of cocycles and extensions of mappings. *Ann. Math.* 48:290-320, 1947.
54	**Stone M H.** Applications of the theory of Boolean rings to general topology. *Trans. Amer. Math. Soc.* 41:375-481, 1937.
37	**Stone M H.** The theory of representations for Boolean algebras. *Trans. Amer. Math. Soc.* 40:37-111, 1936.
32	**Wald A.** Sequential tests of statistical hypotheses. *Ann. Math. Statist.* 16:117-86, 1945.
61	**Wiener N.** Generalized harmonic analysis. *Acta Math.* 55:117-258, 1930.
38	**Yates F.** The analysis of multiple classifications with unequal numbers in the different classes. *J. Amer. Statist. Assn.* 29:51-66, 1934.

to him cannot be viewed as a judgment that he made a significant contribution to mathematics.

Two American mathematicians published highly cited papers in pure mathematics: Nathan Jacobson (b. 1910), Yale University, New Haven, Connecticut, and Norbert Wiener (b. 1894–d. 1964), Massachusetts Institute of Technology, Cambridge. Wiener later became well known to the scientific public for his work in communication theory. Jacobson's work is familiar only to mathematical experts. Curiously, neither is given much attention in works on the history of modern mathematics—perhaps Jacobson's contribution is considered too specialized, Wiener's too "applied." In these cases, the *SCI* helps the historian by calling attention to significant publications that might otherwise be overlooked.

Jacobson's two papers in 1945 presented major advances in abstract algebra, especially the theory of associative rings.[8] He introduced what is now called the "Jacobson radical" of a ring, defined as "the ideal J(A) of an associative ring A which satisfies the following two requirements: 1) J(A) is the largest quasi-regular ideal in A; 2) the quotient ring A_q=A/J(A) contains no nonzero quasi-regular ideals."[9] Based on this concept, the "Jacobson ring" is defined as "a commutative ring with unit element in which any prime ideal is the intersection of the maximal ideals containing it, i.e., a ring any integral quotient ring of which has a zero Jacobson radical."[10] These ideas were

Table 3: Chronologic distribution of publication dates for the 20 mathematics papers most cited in the 1945-1954 *SCI®* cumulation.

Publication Year	Number of Papers
1930-1934	2
1935-1939	6
1940-1944	2
1945-1949	10

further developed in books by Jacobson and others.[11-13]

Wiener has described the circumstances of his work on generalized harmonic analysis, leading to his 1930 paper on that topic, in his autobiography.[14] Harmonic analysis is the decomposition of time-dependent physical processes or mathematical functions into components with different frequencies, pioneered by the French mathematician Joseph Fourier at the beginning of the nineteenth century. The original stimulus for Wiener's work came from problems in electrical engineering. He was able to develop a rigorous theory based on modern mathematical techniques. His interest in practical applications led him to promote the harmonic analysis of time series as a key to many problems in science and engineering.[15,16]

Seventeen of the 20 papers in Table 2 listed one author, and the remaining three have two authors each. Sixteen authors were based at institutions located in the US, and two each were based in the UK and Japan. Table 3 shows the chronologic distribution of publication dates, and Table 4 lists the journals that published the 20 most-cited mathematics articles in the 1945-1954 *SCI*.

The Most Influential Mathematics Publications

Table 5 lists 25 books considered "influential" by the mathematician Paul Richard Halmos, University of Santa Clara, California.[17] Also shown is the number of citations they received in the 1945-1954 *SCI*. There is no algorithm for selecting mathematics books from the ISI database, so it is possible that there are other mathematics books even more highly cited than these.

The leading *Citation Classic®* located with the help of the Halmos list is *A Course of Modern Analysis* by the British mathematician Edmund Taylor Whittaker (b. 1873–d. 1956). Whittaker was a specialist in differential equations and was known for his discovery of integral representations of solutions of Laplace's equation, including the Legendre and Bessel functions. He later wrote a major treatise on the history of optics and electromagnetism.[18] When Whittaker first published *A Course of Modern Analysis* in 1902, it was, according to biographer Daniel Martin, University of Glasgow, UK, "the first book in English to present the theory of functions of a complex variable at a level suitable for undergraduate and beginning graduate students."[19]

George Neville Watson (b. 1886–d. 1965), a British mathematician who was an expert on complex variable theory, collaborated on the preparation of the expanded second edition that appeared in 1915.[20,21] The book became a standard reference work for the properties of special functions and techniques used in mathematical physics. The various editions were cited 420 times in the period 1945-1954, more than the books by Banach and Harald Cramér, University of Stockholm, Sweden, the only mathematics publications on the first list of 52 most-cited publications for that period.[1] But it ap-

Table 4: The journals that published the 20 most-cited mathematics papers. The numbers in parentheses are the 1989 impact factors for the journals. Data were taken from the 1989 *JCR®*. The figures at the right indicate how many papers from each journal appear in Table 2.

Journal	Number of Papers
Ann. Math. (2.01)	4
*Ann. Math. Statist. (N/A)	3
J. Amer. Statist. Assn. (1.17)	3
Quart. Appl. Math. (0.48)	3
Trans. Amer. Math. Soc. (0.54)	3
Amer. J. Math. (0.55)	2
Acta Math. (0.96)	1
J. Roy. Statist. Soc. Ser. B Metho. (1.15)	1

*Divided in 1973 into Ann. Probab. (0.69) and Ann. Statist. (0.97)

Table 5: Mathematics books published before 1955, from a list of books that P.R. Halmos considered influential (see reference 17). Citation totals include 1945-1954 references to all editions and translations. Publication years shown are those given by Halmos; other bibliographic data are taken from the *National Union Catalog*. Books are listed in alphabetic order by first author. A = 1945-1954 citations.

A	Bibliographic Data
167	**Banach S.** *Théorie des opérations linéaires* (Theory of linear operations). Warsaw, Poland: Z subwencji Funduszu kultury narodowej, 1932. 254 p.
46	**Birkhoff G.** *Lattice theory.* New York: American Mathematical Society, 1940. 155 p.
37	**Bôcher M & Duval E P R.** *Introduction to higher algebra.* New York: Macmillan, 1907. 321 p.
0	**Bohnenblust H F.** *Lectures by H.F. Bohnenblust on theory of functions of real variables, 1936-1937.* Ann Arbor, MI: Edwards, 1937. 132 p.
1	**Burington R S & Torrance C C.** *Higher mathematics with applications to science and engineering.* New York: McGraw-Hill, 1939. 844 p.
21	**Carathéodory C.** *Vorlesungen über reelle Funktionen* (Treatise on real functions). Leipzig, Germany: Teubner, 1918. 718 p.
24	**Courant R & McShane E J.** *Vorlesungen über differential und Integralrechnung* (Differential and integral calculus). (McShane E J, trans.) New York: Nordemann, 1938. 2 vols.
1	**Dickson L E.** *Modern algebraic theories.* New York: Sanborn, 1926. 276 p.
8	**Granville W A & Smith P F.** *Elements of the differential and integral calculus.* Boston, MA: Ginn, 1904. 463 p.
14	**Graves L M.** *The theory of functions of real variables.* New York: McGraw-Hill, 1946. 300 p.
4	**Halmos P R.** *Finite dimensional vector spaces.* Princeton, NJ: Princeton University Press, 1942. 196 p.
5	**Hardy G H.** *A course of pure mathematics.* Cambridge, UK: Cambridge University Press, 1908. 428 p.
18	**Hausdorff F.** *Grundzüge der Mengenlehre* (Foundations of set theory). Leipzig, Germany: Veit, 1914. 476 p.
2	**Kleene S C.** *Introduction to metamathematics.* New York: Van Nostrand, 1952. 550 p.
6	**Knopp K.** *Funktionentheorie* (Function theory). Berlin, Germany: de Gruyter, 1930. 2 vols.
23	**Kolmogoroff A N.** *Grundbegriffe der Wahrscheinlichkeitsrechnung* (Foundations of the theory of probability). Berlin, Germany: Springer, 1933. 62 p.
0	**Landau E.** *Grundlagen der Analysis* (Foundations of analysis). Leipzig, Germany: Akademische Verlagsgesellschaft, 1930. 134 p.
30	**Lefschetz S.** *Algebraic topology.* New York: American Mathematical Society, 1942.
108	**Saks S & Banach S.** *Theory of the integral.* Warsaw, Poland: Z subwencji Funduszu kultury narodowej, 1937. 347 p.
1	**Siegel C L & Bellman R.** *Transcendental numbers.* Princeton, NJ: Princeton University Press, 1947. 73 p.
24	**Stone M H.** *Linear transformations in Hilbert space and their applications to analysis.* New York: American Mathematical Society, 1932. 622 p.
2	**Townsend E J.** *Functions of a complex variable.* New York: Holt, 1915. 384 p.
17	**Tukey J W.** *Convergence and uniformity in topology.* Princeton, NJ: Princeton University Press, 1940. 90 p.
135	**van der Waerden B L, Artin E & Noether E.** *Moderne Algebra* (Modern algebra). Berlin, Germany: Springer, 1931. 2 vols.
420	**Whittaker E T.** *A course of modern analysis.* Cambridge, UK: University Press, 1902. 378 p.

peared in several editions and reprintings, no one of which received enough citations to put it on that list as a separate publication.

Astronomy and the Earth Sciences

Next week's essay will examine 22 astronomy journal articles and 20 earth-sciences papers that were most cited in the 1945-1954 *SCI*. These lists will be compared with publications considered influential or important by scientists and historians of science. In addition various trends, achievements, and researchers represented in these lists will be highlighted.

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18. ————. *A history of the theories of aether and electricity.* Los Angeles, CA: Tomash Publishers, 1987. 2 vols.
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