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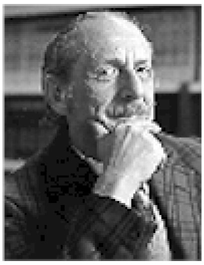

Current Comments®

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The Most-Cited Physical-Sciences Publications in the 1945-1954 *Science Citation Index*. Part 2. Twenty *Citation Classics* in Mathematics

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When I first reported to *Current Contents*® readers that ISI® had compiled the *Science Citation Index*® (*SCI*®) cumulation for 1945-1954, I indicated that it would serve well the growing community of science historians.[1] Indeed, the cumulation has enabled us for the first time to identify the most-cited publications in the postwar decade, a crucial time of rapid growth and development in science and technology.

Last year Bernard Dixon, contributing editor to *Bio/Technology* and former editor of *New Scientist*, discussed the 102 life-sciences papers that were highly cited during this period.[2,3] More recently, the 52 most-cited physical-sciences publications in the 1945-1954 *SCI* were examined by Stephen G. Brush, Department of History and Institute for Physical Science and Technology, University of Maryland, College Park.[4] Since virtually all of these *Citation Classics*® were in chemistry and physics, he requested additional lists of high impact works in mathematics, astronomy, and the earth sciences. In the essay that follows, Brush continues his discussion by examining the 20 most-cited mathematics papers. Next week, he will conclude with a look at 42 *Citation Classics* in astronomy and the earth sciences.

Brush raises a question that often comes up whenever we publish undifferentiated lists of papers ranked by citations: Are these most-cited articles the most "influential"? We have consistently and repeatedly stated that citations alone do not necessarily indicate importance, quality, or influence. We instead prefer to use the more neutral term "impact." That is, citations simply indicate that the cited work has been used in some way by the author referring to it. It would be simplistic, if not absurd, to argue that the importance of research can be measured solely on the basis of citation frequencies and ranks thereof.

Brush attempts to answer this question by comparing the lists of most-cited articles with subjective judgments, such as the award of prestigious prizes, or the opinions of historians of science. As he noted in the first part of his essay, 48 percent of the most-cited physics publications included an author who had won the Nobel Prize. For the high impact chemistry publications, this figure was 40 percent.[4] However, he also observes that these were not necessarily the works for which the authors were honored by the prize, a point I have stated previously.[5]

For the 20 most-cited mathematics articles presented here, Brush uses the Fields Medal as an independent measure of "influence." The Fields Medal is awarded quadrennially by the International Congress of Mathematicians and is widely regarded as equivalent in prestige to the Nobel Prizes. Brush found that no Fields Medal winners were among the authors of these 20 high impact mathematics articles. On this basis, he concludes that "the most-cited publications in mathematics do not contain the most important research," and that "the most important research in mathematics, as judged by awards of the Fields Medal, is not highly cited."

This conclusion is perhaps premature since it is based on a rather small sample

Table 1: The top 100 mathematicians most cited in 1978 and 1979. Asterisks (*) indicate Fields Medal winners. A = citations from the math core journals in 1978 and 1979. B = total citations from *SCI*® journals in 1978 and 1979.

A	B	
164	202	**Adams, John Frank** University of Manchester Manchester, UK
164	265	**Agmon, Shmuel** Hebrew University Jerusalem, Israel
117	171	***Ahlfors, Lars Valerian** Harvard University Cambridge, MA
108	127	**Alfsen, Erik Magnus** University of Oslo Oslo, Norway
140	157	**Artin, Michael** Massachusetts Institute of Technology Cambridge, MA
166	166	**Aschbacher, Michael** California Institute of Technology Pasadena, CA
252	444	***Atiyah, Michael Francis** Oxford University Oxford, UK
126	136	**Auslander, Maurice** Brandeis University Waltham, MA
222	255	**Bass, Hyman** Columbia University New York, NY
110	718	**Bellman, Richard Ernest** University of Southern California Los Angeles, CA
131	333	**Berge, Claude Jacques** CNRS Paris, France
129	338	**Birkhoff, Garrett** Harvard University Cambridge, MA
297	387	**Borel, Armand** Institute for Advanced Study Princeton, NJ
517	723	**Bourbaki, Nicolas** France
137	167	**Brauer, Richard Dagobert** Harvard University Cambridge, MA
115	120	**Bredon, Glen E.** Rutgers University New Brunswick, NJ
173	342	**Brezis, Haim** University of Paris VI Paris, France
190	250	**Browder, Felix Earl** University of Chicago Chicago, IL
101	140	**Calderon, Alberto Pedro** University of Chicago Chicago, IL
179	207	**Carlitz, Leonard** Duke University Durham, NC
167	236	**Cartan, Henri Paul** University of Paris XI Paris, France
131	178	**Clifford, Alfred Hoblitzelle** Tulane University New Orleans, LA
112	147	**Cohn, Paul Moritz** University of London London, UK
103	650	**Courant, Richard** New York University New York, NY
128	159	**Curtis, Charles Whittlesey** University of Oregon Eugene, OR
145	170	***Deligne, Pierre** Institut des Hautes Etudes Scientifiques Bures-sur-Yvette, France
148	263	**Dieudonne, Jean Alexandre** University of Nice Nice, France
346	435	**Dixmier, Jacques** University of Paris VI Paris, France
111	137	**Douglas, Ronald George** State University of New York Stony Brook, NY
324	514	**Dunford, Nelson** Yale University New Haven, CT
101	824	**Erdelyi, Arthur** University of Edinburgh Edinburgh, UK
295	485	**Erdos, Paul** Hungarian Academy of Sciences Budapest, Hungary
119	125	**Feit, Walter** Yale University New Haven, CT
161	356	**Friedman, Avner** Northwestern University Evanston, IL
177	220	**Fuchs, Laszlo** Tulane University New Orleans, LA
207	748	**Gelfand, Izrail Moiseevich** Mathematics Institute USSR Academy of Sciences Moscow, USSR
109	112	**Glauberman, George Isaac** University of Chicago Chicago, IL
103	269	**Gokhberg, Israel T͡Sudikovich** Tel Aviv University Tel Aviv, Israel
293	321	**Gorenstein, Daniel** Rutgers University New Brunswick, NJ
126	131	**Grauert, Hans** University of Gottingen Gottingen, FRG
479	560	***Grothendieck, Alexandre** University of Montpellier II Montpellier, France
106	369	**Hale, Jack Kenneth** Brown University Providence, RI
147	199	**Hall, Marshall** California Institute of Technology Pasadena, CA
207	492	**Halmos, Paul Richard** Indiana University Bloomington, IN
201	483	**Harary, Frank** University of Michigan Ann Arbor, MI
284	461	**Hardy, Godfrey Harold** University of Cambridge Cambridge, UK
161	217	**Harish-Chandra** Institute for Advanced Study Princeton, NJ
138	235	**Hartman, Philip** Johns Hopkins University Baltimore, MD
141	162	**Hartshorne, Robert Cope** University of California Berkeley, CA
125	138	**Hasse, Helmut** University of Hamburg Hamburg, FRG
155	237	**Helgason, Sigurdur** Massachusetts Institute of Technology Cambridge, MA
139	157	**Herstein, Israel N.** University of Chicago Chicago, IL
193	257	**Hewitt, Edwin** University of Washington Seattle, WA
154	309	**Hille, Einar** University of California La Jolla, CA
119	167	**Hilton, Peter John** Case Western Reserve University Cleveland, OH
115	152	**Hirzebruch, Friedrich** University of Bonn Bonn, FRG
334	491	***Hormander, Lars Volter** University of Lund Lund, Sweden
178	189	**Huppert, Bertram** University of Mainz Mainz, FRG
111	123	**Iwasawa, Kenkichi** Princeton Univ. Princeton, NJ
261	343	**Jacobson, Nathan** Yale University New Haven, CT
225	285	**Kaplansky, Irving** University of Chicago Chicago, IL
131	638	**Karlin, Samuel** Stanford University Stanford, CA
343	646	**Kato, Tosio** University of California Berkeley, CA
144	201	**Kobayashi, Shoshichi** University of California Berkeley, CA
131	263	**Krasnoselskii, Mark Aleksandrovich** Moscow Control Problems Institute Moscow, USSR
136	212	**Kuratowski, Kazimierz** Warsaw University Warsaw, Poland
124	285	**Ladyzhenskaia, Olga Aleksandrovna** Leningrad University Leningrad, USSR

A	B	
227	311	**Lang, Serge** Yale University New Haven, CT
108	404	**Lax, Peter David** New York University New York, NY
150	167	**Lindenstrauss, Joram** Hebrew University Jerusalem, Israel
210	552	**Lions, Jacques-Louis** College de France Paris, France
112	140	**Lorentz, George G.** University of Texas Austin, TX
137	249	**Mackey, George Whitelaw** Harvard University Cambridge, MA
125	188	**MacLane, Saunders** University of Chicago Chicago, IL
105	133	**May, J. Peter** University of Chicago Chicago, IL
415	542	***Milnor, John Willard** Institute for Advanced Study Princeton, NJ
206	228	***Mumford, David Bryant** Harvard University Cambridge, MA
119	147	**Nagata, Masayoshi** Kyoto University Kyoto, Japan
113	269	**Polya, George** Stanford University Stanford, CA
137	164	***Quillen, Daniel G.** Massachusetts Institute of Technology Cambridge, MA
109	370	**Rockafellar, Ralph Tyrrell** University of Washington Seattle, WA
224	349	**Rudin, Walter** University of Wisconsin Madison, WI
110	138	**Sakai, Shoichiro** Nihon University Tokyo, Japan
111	153	**Sato, Mikio** Kyoto University Kyoto, Japan
133	172	**Schaefer, Helmut H.** University of Tubingen Tubingen, FRG
125	325	***Schwartz, Laurent** Ecole Polytechnique Plaiseau, France
390	463	***Serre, Jean-Pierre** College de France Paris, France
160	185	**Shimura, Goro** Princeton University Princeton, NJ
129	196	**Siegel, Carl Ludwig** University of Gottingen Gottingen, FRG
102	126	**Spanier, Edwin Henry** University of California Berkeley, CA
253	394	**Stein, Elias M.** Princeton University Princeton, NJ
104	112	**Swan, Richard Gordon** University of Chicago Chicago, IL
137	339	**Titchmarsh, Edward Charles** Oxford University Oxford, UK
104	190	**Tutte, William Thomas** University of Waterloo Waterloo, Canada
145	162	**Wall, Charles Terence Clegg** University of Liverpool Liverpool, UK
187	248	**Weil, Andre** Institute for Advanced Study Princeton, NJ
110	487	**Weyl, Hermann** Institute for Advanced Study Princeton, NJ
131	190	**Whitney, Hassler** Institute for Advanced Study Princeton, NJ
190	214	**Zariski, Oscar** Harvard University Cambridge, MA
213	290	**Zygmund, Antoni** University of Chicago Chicago, IL

Table 2: The next 103 mathematicians most cited in 1978 and 1979. Asterisks (*) indicate Fields Medal winners. A=citations from the math core journals in 1978 and 1979. B=total citations from *SCI*® journals in 1978 and 1979.

A	B	
70	90	**Amann, Herbert** University of Zurich Zurich, Switzerland
68	76	**Amitsur, Shimshon A.** Hebrew University Jerusalem, Israel
71	80	**Andreotti, Aldo** University of Strasbourg Strasbourg, France
91	387	**Arnold, Victor Igorevich** University of Moscow Moscow, USSR
94	123	**Artin, Emil** University of Hamburg Hamburg, FRG
74	98	**Arveson, William Barnes** University of California Berkeley, CA
83	90	**Auslander, Louis** City University of New York New York, NY
99	154	**Bers, Lipman** Columbia University New York, NY
93	104	**Bing, R.H.** University of Texas Austin, TX
72	105	**Boas, Ralph P.** Northwestern University Evanston, IL
93	100	**Bonsall, Frank Featherstone** University of Edinburgh Edinburgh, UK
79	118	**Bott, Raoul** Harvard University Cambridge, MA
77	91	**Bousfield, Aldridge Knight** University of Illinois Chicago, IL
83	109	**Bowen, Robert E.** University of California Berkeley, CA
89	95	**Browder, William** Princeton University Princeton, NJ
69	76	**Carleson, Lennart Axel Edvard** Mittag-Leffler Institute Djursholm, Sweden
100	111	**Cassels, John William Scott** University of Cambridge Cambridge, UK
87	142	**Chern, Shiing-Shen** University of California Berkeley, CA
93	133	**Chevalley, Claude** University of Paris VIII Paris, France
76	194	**Coddington, Earl A.** University of California Los Angeles, CA
78	87	**Conner, Pierre Euclide** Louisiana State University Baton Rouge, LA
83	113	**Connes, Alain** Institut des Hautes Etudes Scientifiques Bures-sur-Yvette, France
78	119	**Coxeter, Harold Scott MacDonald** University of Toronto Toronto, Canada
74	162	**Crandall, Michael G.** University of Wisconsin Madison, WI
87	91	**Day, Mahlon M.** University of Illinois Urbana, IL
69	89	**Demazure, Michael** Ecole Polytechnique Plaiseau, France
91	109	**Dickson, Leonard E.** University of Chicago Chicago, IL
70	92	**Diestel, Joseph** Kent State University Kent, OH
69	90	**Dold, Albrecht** University of Heidelberg Heidelberg, FRG
100	171	**Eilenberg, Samuel** Columbia University New York, NY
81	93	**Everitt, William Norrie** University of Dundee Dundee, UK
74	104	**Federer, Herbert** Brown University Providence, RI
100	121	***Fefferman, Charles Louis** Princeton University Princeton, NJ
91	885	**Feller, William** Princeton University Princeton, NJ
70	76	**Fox, Ralph H.** Princeton University Princeton, NJ

A	B	
86	97	**Frohlich, Albrecht** University of London London, UK
73	91	**Gabriel, Pierre** University of Zurich Zurich, Switzerland
75	90	**Gamelin, Theodore W.** University of California Los Angeles, CA
80	103	**Gillman, Leonard** University of Texas Austin, TX
78	79	**Goldschmidt, David M.** University of California Berkeley, CA
68	97	**Gratzer, George** *University of Manitoba* Winnipeg, Canada
94	109	**Griffiths, Phillip A.** Harvard University Cambridge, MA
97	115	**Gunning, Robert Clifford** Princeton University Princeton, NJ
94	117	**Hall, Philip** University of Cambridge Cambridge, UK
69	73	**Hayman, Walter Kurt** University of London London, UK
82	88	**Higman, Graham** Oxford University Oxford, UK
94	112	***Hironaka, Heisuke** Harvard University Cambridge, MA
76	143	**Hirsch, Morris William** University of California Berkeley, CA
87	119	**Hochschild, Gerhard P.** University of California Berkeley, CA
81	90	**Hochster, Melvin** University of Michigan Ann Arbor, MI
81	118	**Hoffman, Kenneth** Hampshire College Amherst, MA
83	100	**Humphreys, James E.** University of Massachusetts Amherst, MA
76	99	**Kadison, Richard Vincent** University of Pennsylvania Philadelphia, PA
96	124	**Kelley, John Le Roy** University of California Berkeley, CA
68	74	**Kervaire, Michel A.** University of Geneva Geneva, Switzerland
90	94	**Knebusch, Manfred** University of Regensburg Regensburg, FRG
82	121	***Kodaira, Kunihiko** University of Tokyo Tokyo, Japan
69	525	**Kolmogorov, Andrey Nikolaevich** USSR Academy of Sciences Moscow, USSR
96	169	**Kostant, Bertram** Massachusetts Institute of Technology Cambridge, MA
93	118	**Kothe, Gottfried** University of Frankfurt Frankfurt, FRG
92	130	**Krein, Mark G.** Institute of Physical Chemistry Odessa, USSR
82	88	**Lambek, Joachim** McGill University Montreal, Canada
78	98	**Luxemburg, Wilhelmus Anthonius Josephus** California Institute of Technology Pasadena, CA
99	259	**Magnus, Wilhelm** *Polytechnic Institute of New York* New York, NY
92	135	**Michael, Ernest Arthur** University of Washington Seattle, WA
85	102	**Moore, Calvin C.** University of California Berkeley, CA
78	90	**Morrey, Charles Bradfield** University of California Berkeley, CA
89	294	**Neumann, John Von** Institute for Advanced Study Princeton, NJ
72	113	**Nikolskii, Sergei Mikhailovich** Steklov Institute of Mathematics Moscow, USSR
68	85	**Nussbaum, Roger David** Rutgers University New Brunswick, NJ
74	124	**Palais, Richard Sheldon** Brandeis University Waltham, MA
74	82	**Pederson, Gert Kjaergard** University of Copenhagen Copenhagen, Denmark
82	113	**Peetre, Jaak** University of Lund Lund, Sweden
76	79	**Pelczynski, Aleksander** Polish Academy of Sciences Warsaw, Poland
71	96	**Pietsch, Albrecht** University of Jena Jena, GDR
71	225	**Reed, Michael Charles** Duke University Durham, NC
74	151	**Riesz, Friedrich** University of Budapest Budapest, Hungary
95	111	**Rosenthal, Haskell P.** University of Texas Austin, TX
70	109	**Schechter, Murray** Lehigh University Bethlehem, PA
83	182	**Schoenberg, Isaac J.** University of Wisconsin Madison, WI
72	86	**Segal, Graeme Bryce** Oxford University Oxford, UK
97	271	**Segal, Irving Ezra** Massachusetts Institute of Technology Cambridge, MA
81	98	**Sierpinski, Waclaw F.** Warsaw University Warsaw, Poland
99	287	**Simon, Barry Martin** California Institute of Technology Pasadena, CA
98	225	***Smale, Stephen** University of California Berkeley, CA
72	77	**Stallings, John Robert** University of California Berkeley, CA
82	146	**Steenrod, Norman E.** Princeton University Princeton, NJ
70	70	**Steinberg, Robert** University of California Los Angeles, CA
87	106	**Sullivan, Dennis** Institut des Hautes Etudes Scientifiques Bures-sur-Yvette, France
73	73	**Suzuki, Michio** University of Illinois Urbana, IL
72	232	**Szego, Gabor** Stanford University Stanford, CA
92	172	**Szokefalvi-Nagy, Bela** University of Szeged Szeged, Hungary
88	127	**Takesaki, Masamichi** University of California Los Angeles, CA
95	101	**Tate, John T.** Harvard University Cambridge, MA
80	85	***Thompson, John G.** University of Cambridge Cambridge, UK
74	74	**Timmesfeld, Frantz G.** University of Cologne Cologne, FRG
88	102	**Tits, Jacques** College de France Paris, France
83	130	**Treves, Francois** Rutgers University New Brunswick, NJ
71	83	**Triebel, Hans** University of Jena Jena, GDR
80	82	**Waldhausen, Friedhelm** University of Bielefeld Bielefeld, FRG
80	108	**Warner, Garth W.** University of Washington Seattle, WA
75	87	**Wolf, Joseph Albert** University of California Berkeley, CA
100	189	**Yosida, Kosaku** Gakushuin University Tokyo, Japan

of just 20 most-cited journal articles. A more adequate sample would be on the order of 1,000-1,500 articles. But even this might not be sufficient unless books were also included. During the period examined here, 1945-1954, books may have been equally or more important than journals as a means of communicating mathematical research.

An alternative approach would be to examine a list of most-cited authors rather than highest impact publications to see whether Fields Medal winners are highly cited. In 1982 we identified the 200 "pure" mathematicians who were most cited in 1978 and 1979.[6] Through 1978, 24 individuals had received the Fields Medal. Fifteen of them (63 percent) appeared on the list of 200 most-cited mathematicians. They are identified by asterisks in Tables 1 and 2, which are reprinted from that study.

Another author on the list, Alain Connes, Institut des Hautes Études Scientifiques, Bures-sur-Yvette, France, won the Fields Medal in 1982. Including Connes, the 16 Fields Medal winners were cited 3,137 times in a set of 64 "core" mathematics journals and 4,335 times in all *SCI* journals in 1978 and 1979. Thus, the average number of citations per author was 196 and 271, respectively. In comparison, the 187 non-Fields winners received 22,578 core mathematics citations and 38,281 all *SCI* citations, yielding per author averages of 121 and 205, respectively. These data indicate that Fields Medal winners are indeed highly cited and have higher average impacts than the non-winners.

It is worthwhile to note a few limitations of the Fields Medal. The judges tend to limit awards to mathematicians who are no more than 40 years old. Theoretically, those who make major contributions to mathematics in later life would not necessarily be recognized by a Fields Medal. This is in contrast to the Nobel Prizes, for which researchers are eligible throughout their lifetimes.

Also, the Fields Medal judges might sometimes seem to make arbitrary decisions on who is a "mathematician." This was most recently illustrated in the 1990 awards. Shigefumi Mori, Research Institute of Mathematical Sciences, Kyoto University, Japan, was honored for developing the classification of complex algebraic varieties—presumably, a contribution to "pure" mathematics.[7] But the other winners could be considered mathematicians or theoretical physicists.

Vladimir G. Drinfeld, Institute for Low Temperature Physics and Engineering, Kharkov, USSR, was recognized for his research in algebraic geometry, number theory, quantum groups, and other mathematical specialties that are related to theoretical physics. Vaughan F.R. Jones, University of California, Berkeley, worked on knot theory, a branch of topology that has applications both to elementary particles and DNA. Edward Witten, Institute for Advanced Study, Princeton, New Jersey, was honored for his fundamental contributions to the development of string theory, which might shed light on the relationship between gravity and other natural forces.[7]

Witten has consistently appeared in our recent studies of the most-cited physical-sciences papers.[8,9] He also appeared on a "shortlist" of 12 researchers forecasted to win the Nobel Prize in physics that appeared in *The Scientist*®.[10] Similar lists of "nominees" for the Nobel Prizes in chemistry as well as medicine or physiology are being prepared as we go to press. Last year *The Scientist* identified 20 likely candidates for the 1989 Nobel Prize in medicine or physiology, two of whom did go on to win it—J. Michael Bishop and Harold E. Varmus.[11]

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The Most-Cited Physical-Sciences Publications in the 1945-1954 *Science Citation Index*. Part 2. Mathematics

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Back to Introduction

This essay examines 20 highly cited papers in mathematics, based on the *Science Citation Index*® cumulation for 1945-1954. Next week 42 most-cited papers in astronomy and the earth sciences will be examined. These papers are compared with other publications (including some highly cited books) considered important by scientists and historians of science. The essay discusses some of the major trends, achievements, and researchers in mathematics in the period including World War II.

Introduction: Finding Highly Cited Publications in Small Fields

In Part 1 of this essay, I discussed 52 highly cited publications in the physical sciences, based on the *Science Citation Index*® (*SCI*®) cumulation for 1945-1954.[1] That list was composed almost entirely of publications in chemistry (25) and physics (25); there were only two in mathematics, and none in astronomy or the earth sciences. Just as one cannot ignore the physical sciences merely because they generate fewer citations than the biological sciences,[2] one cannot simply ignore astronomy, the earth sciences, and mathematics merely because they generate fewer citations than physics and chemistry. ISI® has therefore generated additional lists of relatively highly cited papers in these smaller, less-cited fields. In addition, I present lists of publications considered important by scientists or historians of science.

Do Citations Measure Importance? The Case of Mathematics

As noted in Part 1 of this essay, one should not simply rely on citation counts as a measure of the importance or quality of a publication. Rather, it is desirable also to obtain the independent judgments of the scientific community—for example, as indicated by Nobel Prizes—or of historians of science. Thus, 48 percent of the most-cited physics publications and 40 percent of the most-cited chemistry publications were authored or coauthored by a Nobel laureate, although those publications were not necessarily the work for which they received the Nobel Prize.[1]

For mathematics, the closest equivalent to the Nobel Prize is the Fields Medal, awarded at the quadrennial International Congress of Mathematicians, beginning in 1936. No medals were given between 1936 and 1950; the medals awarded in 1936,

Table 1: Winners of the Fields Medal in mathematics, awarded at the International Congress of Mathematicians in 1936, 1950, and 1954, and their areas of research. Medalists are listed in alphabetic order. Dates in parentheses in the "Research Area" column give the time period when the medal-winning work was done.

Medalist	Year Awarded	Research Area
Ahlfors L	1936	Complex-variable theory, quasiconformal mappings, Riemann surfaces, meromorphic functions (1920s, 1930s).
Douglas J	1936	Solved Plateau problem (minimal surface) (1931).
Kodaira K	1954	Harmonic integrals and harmonic forms with application to Kahlerian and algebraic varieties (1944-1953).
Schwartz L	1950	Theory of distributions (1945-1951).
Selberg A	1950	Prime number theorem (1948-1949), Riemann zeta function (1940s).
Serre J-P	1954	Complex variables, cohomology in a complex-analytic sheaf (1950-1951).

1950, and 1954 were for research by six mathematicians published in the period from about 1930 to about 1952. These are listed in Table 1.

Table 2 presents 20 mathematics journal articles that were most cited in the 1945-1954 *SCI*. Comparing both tables, one can see that none of the Fields Medal winners appear as authors of the 20 most-cited mathematics articles during this period. The most-cited journal article by a Fields winner is by Jean-Pierre Serre, College of France, Paris.[3] Its 27 citations from 1945 to 1954, however, are too few to put it on the list of 20 most-cited mathematics papers, which were cited at least 30 times. Citations for the 1958 Fields Medal winners (Klaus F. Roth, University of London, UK, and René Thom, University of Strasbourg, France) were even fewer, so including them would not make any difference to our conclusion: the most important research in mathematics, as judged by awards of the Fields Medal, is not highly cited, and the most-cited publications in mathematics do not contain the most important research.

Some mathematicians would undoubtedly argue that Stefan Banach's (University of Lvov, USSR) *Théorie des opérations linéaires* is a counterexample to this generalization, since it showed up on the list of 52 most-cited physical-sciences papers and is generally regarded as a report of important original research.[1,4] Nevertheless, it did not win the 1936 Fields Medal for which it was presumably eligible.

The Most-Cited Mathematics Articles

The research areas of the Fields Medal winners and the most-cited papers published in mathematics journals indicate trends in pure mathematics during the 1930s and 1940s. Abstract algebra and topology were the most popular subjects. As Jean Dieudonné, University of Nice, France, expressed it in his survey of modern mathematics, the emphasis was on studying the structure rather than the content of mathematical objects.[5] Most of the highly cited mathematicians are listed as "originators" of one or more of the research specialties described by Dieudonné.[6]

The most-cited mathematics article is on statistics and was authored by Henry B. Mann and D.R. Whitney, Ohio State University, Columbus. Most of its 109 citations from 1945 to 1954 are from biological and medical journals, so one may question whether it should be included in a list of highly cited *physical-sciences* publications. Mann has described its origin in the problem of testing a drug that was supposed to protect against the common cold.[7]

One of the most-cited papers in mathematics journals was by Milton Friedman (b. 1912), then with the National Resources Committee, Washington, DC, who won the 1976 Nobel Prize for economics. Presumably, his 1937 paper on the use of rank ordering in statistical analysis was only a small part of the body of work for which he was honored, and the award of the Nobel Prize

Table 2: The 20 most-cited papers from mathematics journals covered in the 1945-1954 *SCI*® cumulation. Papers are listed in alphabetic order by first author. A = total number of 1945-1954 citations.

A	Bibliographic Data
30	**Bartlett M S.** On the theoretical specification and sampling properties of autocorrelated time-series. *J. Roy. Statist. Soc. Ser. B Metho.* 8:27-41, 1946.
31	**Berkson J.** Application of the logistic function to bio-assay. *J. Amer. Statist. Assn.* 39:357-65, 1944.
33	**Friedman M.** The use of ranks to avoid the assumption of normality implicit in the analysis of variance. *J. Amer. Statist. Assn.* 32:675-701, 1937.
32	**Iwasawa K.** On some types of topological groups. *Ann. Math.* 50:507-58, 1949.
61	**Jacobson N.** The radical and semi-simplicity for arbitrary rings. *Amer. J. Math.* 67:300-20, 1945.
38	**Jacobson N.** Structure theory of simple rings without finiteness assumptions. *Trans. Amer. Math. Soc.* 57:228-45, 1945.
30	**Kakutani S.** Concrete representation of abstract (*M*)-spaces (A characterization of the space of continuous functions). *Ann. Math.* 42:994-1024, 1941.
31	**King R & Middleton D.** The cylindrical antenna; current and impedance. *Quart. Appl. Math.* 3:302-35, 1946.
35	**Lin C C.** On the stability of two-dimensional parallel flows. Part I.—General theory. *Quart. Appl. Math.* 3:117-42, 1945.
109	**Mann H B & Whitney D R.** On a test of whether one of two random variables is stochastically larger than the other. *Ann. Math. Statist.* 18:50-60, 1947.
30	**Middleton D.** Some general results in the theory of noise through non-linear devices. *Quart. Appl. Math.* 5:445-98, 1947.
32	**Murnaghan F D.** Finite deformations of an elastic solid. *Amer. J. Math.* 59:235-60, 1937.
41	**Murray F J & von Neumann J.** On rings of operators. *Ann. Math.* 37:116-229, 1936.
33	**Neyman J.** On a class of "contagious" distributions, applicable in entomology and bacteriology. *Ann. Math. Statist.* 10:35-57, 1939.
32	**Steenrod N E.** Products of cocycles and extensions of mappings. *Ann. Math.* 48:290-320, 1947.
54	**Stone M H.** Applications of the theory of Boolean rings to general topology. *Trans. Amer. Math. Soc.* 41:375-481, 1937.
37	**Stone M H.** The theory of representations for Boolean algebras. *Trans. Amer. Math. Soc.* 40:37-111, 1936.
32	**Wald A.** Sequential tests of statistical hypotheses. *Ann. Math. Statist.* 16:117-86, 1945.
61	**Wiener N.** Generalized harmonic analysis. *Acta Math.* 55:117-258, 1930.
38	**Yates F.** The analysis of multiple classifications with unequal numbers in the different classes. *J. Amer. Statist. Assn.* 29:51-66, 1934.

to him cannot be viewed as a judgment that he made a significant contribution to mathematics.

Two American mathematicians published highly cited papers in pure mathematics: Nathan Jacobson (b. 1910), Yale University, New Haven, Connecticut, and Norbert Wiener (b. 1894–d. 1964), Massachusetts Institute of Technology, Cambridge. Wiener later became well known to the scientific public for his work in communication theory. Jacobson's work is familiar only to mathematical experts. Curiously, neither is given much attention in works on the history of modern mathematics—perhaps Jacobson's contribution is considered too specialized, Wiener's too "applied." In these cases, the *SCI* helps the historian by calling attention to significant publications that might otherwise be overlooked.

Jacobson's two papers in 1945 presented major advances in abstract algebra, especially the theory of associative rings.[8] He introduced what is now called the "Jacobson radical" of a ring, defined as "the ideal J(A) of an associative ring A which satisfies the following two requirements: 1) J(A) is the largest quasi-regular ideal in A; 2) the quotient ring $A_q = A/J(A)$ contains no non-zero quasi-regular ideals."[9] Based on this concept, the "Jacobson ring" is defined as "a commutative ring with unit element in which any prime ideal is the intersection of the maximal ideals containing it, i.e., a ring any integral quotient ring of which has a zero Jacobson radical."[10] These ideas were

Table 3: Chronologic distribution of publication dates for the 20 mathematics papers most cited in the 1945-1954 *SCI*® cumulation.

Publication Year	Number of Papers
1930-1934	2
1935-1939	6
1940-1944	2
1945-1949	10

further developed in books by Jacobson and others.[11-13]

Wiener has described the circumstances of his work on generalized harmonic analysis, leading to his 1930 paper on that topic, in his autobiography.[14] Harmonic analysis is the decomposition of time-dependent physical processes or mathematical functions into components with different frequencies, pioneered by the French mathematician Joseph Fourier at the beginning of the nineteenth century. The original stimulus for Wiener's work came from problems in electrical engineering. He was able to develop a rigorous theory based on modern mathematical techniques. His interest in practical applications led him to promote the harmonic analysis of time series as a key to many problems in science and engineering.[15,16]

Seventeen of the 20 papers in Table 2 listed one author, and the remaining three have two authors each. Sixteen authors were based at institutions located in the US, and two each were based in the UK and Japan. Table 3 shows the chronologic distribution of publication dates, and Table 4 lists the journals that published the 20 most-cited mathematics articles in the 1945-1954 *SCI*.

The Most Influential Mathematics Publications

Table 5 lists 25 books considered "influential" by the mathematician Paul Richard Halmos, University of Santa Clara, California.[17] Also shown is the number of citations they received in the 1945-1954 *SCI*. There is no algorithm for selecting mathematics books from the ISI database, so it is possible that there are other mathematics books even more highly cited than these.

The leading *Citation Classic*® located with the help of the Halmos list is *A Course of Modern Analysis* by the British mathematician Edmund Taylor Whittaker (b. 1873–d. 1956). Whittaker was a specialist in differential equations and was known for his discovery of integral representations of solutions of Laplace's equation, including the Legendre and Bessel functions. He later wrote a major treatise on the history of optics and electromagnetism.[18] When Whittaker first published *A Course of Modern Analysis* in 1902, it was, according to biographer Daniel Martin, University of Glasgow, UK, "the first book in English to present the theory of functions of a complex variable at a level suitable for undergraduate and beginning graduate students."[19]

George Neville Watson (b. 1886–d. 1965), a British mathematician who was an expert on complex variable theory, collaborated on the preparation of the expanded second edition that appeared in 1915.[20,21] The book became a standard reference work for the properties of special functions and techniques used in mathematical physics. The various editions were cited 420 times in the period 1945-1954, more than the books by Banach and Harald Cramér, University of Stockholm, Sweden, the only mathematics publications on the first list of 52 most-cited publications for that period.[1] But it ap-

Table 4: The journals that published the 20 most-cited mathematics papers. The numbers in parentheses are the 1989 impact factors for the journals. Data were taken from the 1989 *JCR*®. The figures at the right indicate how many papers from each journal appear in Table 2.

Journal	Number of Papers
Ann. Math. (2.01)	4
*Ann. Math. Statist. (N/A)	3
J. Amer. Statist. Assn. (1.17)	3
Quart. Appl. Math. (0.48)	3
Trans. Amer. Math. Soc. (0.54)	3
Amer. J. Math. (0.55)	2
Acta Math. (0.96)	1
J. Roy. Statist. Soc. Ser. B Metho. (1.15)	1

*Divided in 1973 into Ann. Probab. (0.69) and Ann. Statist. (0.97)

Table 5: Mathematics books published before 1955, from a list of books that P.R. Halmos considered influential (see reference 17). Citation totals include 1945-1954 references to all editions and translations. Publication years shown are those given by Halmos; other bibliographic data are taken from the *National Union Catalog*. Books are listed in alphabetic order by first author. A = 1945-1954 citations.

A	Bibliographic Data
167	**Banach S.** *Théorie des opérations linéaires* (Theory of linear operations). Warsaw, Poland: Z subwencji Funduszu kultury narodowej, 1932. 254 p.
46	**Birkhoff G.** *Lattice theory.* New York: American Mathematical Society, 1940. 155 p.
37	**Bôcher M & Duval E P R.** *Introduction to higher algebra.* New York: Macmillan, 1907. 321 p.
0	**Bohnenblust H F.** *Lectures by H.F. Bohnenblust on theory of functions of real variables, 1936-1937.* Ann Arbor, MI: Edwards, 1937. 132 p.
1	**Burington R S & Torrance C C.** *Higher mathematics with applications to science and engineering.* New York: McGraw-Hill, 1939. 844 p.
21	**Carathéodory C.** *Vorlesungen über reelle Funktionen* (Treatise on real functions). Leipzig, Germany: Teubner, 1918. 718 p.
24	**Courant R & McShane E J.** *Vorlesungen über differential und Integralrechnung* (Differential and integral calculus). (McShane E J, trans.) New York: Nordemann, 1938. 2 vols.
1	**Dickson L E.** *Modern algebraic theories.* New York: Sanborn, 1926. 276 p.
8	**Granville W A & Smith P F.** *Elements of the differential and integral calculus.* Boston, MA: Ginn, 1904. 463 p.
14	**Graves L M.** *The theory of functions of real variables.* New York: McGraw-Hill, 1946. 300 p.
4	**Halmos P R.** *Finite dimensional vector spaces.* Princeton, NJ: Princeton University Press, 1942. 196 p.
5	**Hardy G H.** *A course of pure mathematics.* Cambridge, UK: Cambridge University Press, 1908. 428 p.
18	**Hausdorff F.** *Grundzüge der Mengenlehre* (Foundations of set theory). Leipzig, Germany: Veit, 1914. 476 p.
2	**Kleene S C.** *Introduction to metamathematics.* New York: Van Nostrand, 1952. 550 p.
6	**Knopp K.** *Funktionentheorie* (Function theory). Berlin, Germany: de Gruyter, 1930. 2 vols.
23	**Kolmogoroff A N.** *Grundbegriffe der Wahrscheinlichkeitsrechnung* (Foundations of the theory of probability). Berlin, Germany: Springer, 1933. 62 p.
0	**Landau E.** *Grundlagen der Analysis* (Foundations of analysis). Leipzig, Germany: Akademische Verlagsgesellschaft, 1930. 134 p.
30	**Lefschetz S.** *Algebraic topology.* New York: American Mathematical Society, 1942.
108	**Saks S & Banach S.** *Theory of the integral.* Warsaw, Poland: Z subwencji Funduszu kultury narodowej, 1937. 347 p.
1	**Siegel C L & Bellman R.** *Transcendental numbers.* Princeton, NJ: Princeton University Press, 1947. 73 p.
24	**Stone M H.** *Linear transformations in Hilbert space and their applications to analysis.* New York: American Mathematical Society, 1932. 622 p.
2	**Townsend E J.** *Functions of a complex variable.* New York: Holt, 1915. 384 p.
17	**Tukey J W.** *Convergence and uniformity in topology.* Princeton, NJ: Princeton University Press, 1940. 90 p.
135	**van der Waerden B L, Artin E & Noether E.** *Moderne Algebra* (Modern algebra). Berlin, Germany: Springer, 1931. 2 vols.
420	**Whittaker E T.** *A course of modern analysis.* Cambridge, UK: University Press, 1902. 378 p.

peared in several editions and reprintings, no one of which received enough citations to put it on that list as a separate publication.

Astronomy and the Earth Sciences

Next week's essay will examine 22 astronomy journal articles and 20 earth-sciences papers that were most cited in the 1945-1954 *SCI*. These lists will be compared with publications considered influential or important by scientists and historians of science. In addition various trends, achievements, and researchers represented in these lists will be highlighted.

* * * * *

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11. **Jacobson N.** *Structure of rings.* Providence, RI: American Mathematical Society, 1956. 263 p.
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15. ————. *Cybernetics.* New York: Wiley, 1948. 194 p.
16. **Heims S J.** *John von Neumann and Norbert Wiener: from mathematics to the technologies of life and death.* Cambridge, MA: MIT Press, 1980. 547 p.
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18. ————. *A history of the theories of aether and electricity.* Los Angeles, CA: Tomash Publishers, 1987. 2 vols.
19. **Martin D.** Whittaker, Edmund Taylor. *Dictionary of scientific biography.* New York: Scribner's, 1980. Vol. 14. p. 316-8.
20. **Rankin R A.** Watson, George Neville. *Dictionary of scientific biography.* New York: Scribner's, 1980. Vol. 14. p. 188-9.
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