"current comments"

Highly Cited Works in Mathematics. Part 1. "Pure" Mathematics

November 21, 1973 Number 47

At the end of this essay you will find a list of mathematics books and journal articles most highly cited in the *Science Citation Index®* (*SCI®*) in the period 1961-1972. Since the *SCI* data base now contains more than 30 million citations, the list is significant.

The list that appears this week contains 78 items. Most of the items listed (56) are books. All are concerned, according to criteria described below, with "pure" mathematics. Some mathematicians may justifiably argue that some are not. But the citation record indicates that they have been heavily cited by other mathematicians, and so they are, if not "pure" mathematics, at least of definite interest to the mathematicians.

A second list will appear next week. It contains about the same number of publications from applied mathematics, particularly statistics, biometrics, etc.

Precise categorization is difficult, if not impossible, in this sort of exercise. Where "pure" mathematics ends and "applied" mathematics begins is problematical. Our quantitative criteria may nevertheless have as much validity as any *a priori* qualitative definition. Our list is, in fact, an elementary exercise in numerical taxonomy.

As in other fields, but especially in mathematics, citation frequency must be cautiously interpreted. There are many areas in mathematics, number theory in particular, where the number of people working on a particular problem is so small (only three or four in some cases) that, however distinguished their work, it can never be highly cited.

Similar arguments can perhaps be made for non-mathematics papers, and the large number of esoteric items on this list may seem to belie the premise. In any case, I hope these lists will be useful to mathematicians and bibliographers as an example of *statistical bibliography* (bibliometrics), based on citation analysis. And it certainly ought to suggest to those of us who are not mathematicians something about where the action is and what its impact may be.

To construct the list, we extracted from our *SCI* data base every item cited ten or more times in *any* of the years 1961 through 1972. We then calculated, for each item selected, the total number of citations for the twelve years. Then we ranked them by frequency of citation. From this very large list, we extracted all items that appeared to be books on mathematics and its applications, or articles published in mathematics, statistics, biometrics, and other "applied" mathematics journals.

Recognizing that this procedure might overlook important mathematical and statistical work appearing in multidisciplinary "nonmathematics" journals (*PNAS*, *Doklady*, etc.), we then used a rather rigorous algorithm to find them. We extracted a list of all items cited in 1972 issues of mathematics and statistics journals covered by the *Science Citation Index*.[1] Any item cited in these journals in 1972 at least 4 times, *and cited in common by at least 3 papers in those same 1972 issues,* was considered to be "mathematical." We added it to our "highly cited" mathematics list, if over the period 1961 through 1972, it had been cited 80 or more times. (The requirement of 80 citations was made purely on the basis of space available to print the list in these pages.)

In the frustrating process of categorizing the items retrieved as "pure" or "applied" we were advised by Professor Kenneth O. May of the University of Toronto, a member of the *Current Contents*® Editorial Advisory Board. Our ultimate decision was to list each item on one list or the other. Lack of space prevents me from reporting the many interesting comments Professor May provided on the significance of the items listed. Remarkably, in the majority of cases his independent judgment validated the results of the algorithm described above in distinguishing between "pure" and "applied." (The "pure mathematical" quality of the list presented here may be judged, I believe, from the many times two of the citation totals show no great disparity, the number of times cited by all journals in 1972 and the number of times cited by only mathematical and statistical journals that same year.) The lists are primarily offered, as I have said, as examples of statistical bibliography or bibliometrics--not as examples of mathematical scholarship. We hope, however, that they may be useful in that regard.

1. These journals are listed in: *Science Citation Index 1972 Guide & Journal Lists* (Philadelphia: Institute for Scientific Information, 1973), p. 102, 103. (This *SCI Guide* also appears as front-matter in volume 5 of the *Science Citation Index 1972.*)

A List of Heavily Cited Works in Mathematics, 1961-1972.

Part 1. "Pure" Mathematics

Rank	Total Citations 1961-1972	1972	Selective 1972	Bibliographical Data
(1)	(2)	(3)	(4)	(5)
1.	563	156	(115)	**Feller, W.** *Introduction to probability theory and its applications.* 2 vols., 3rd ed. Wiley, 1968.
2.	518	119	(64)	**Dunford, N. & Schwartz, J.T.** *Linear operators.* 3 pts. Wiley, 1958.
3.	432	109	(65)	**Erdelyi, A.,** ed. *Higher transcendental functions.* 3 vols. McGraw, 1953.
4.	383	38	(10)	**Shannon, C.E.** A mathematical theory of communications. *Bell Syst. Tech. J.* 27:379-423, 1948.
5.	365	155	(4)	**Gradshteyn, I.S. & Tyzhik, I.M.** *Tables of integrals, series, and products.* Academic, 1966.
6.	338	67	(11)	**Fletcher, R. & Powell, M.J.D.** A rapidly convergent descent method for minimization. *Computer J.* 6:163-168, 1963.
7.	320	75	(75)	**Doob, J.L.** *Stochastic processes.* Wiley, 1953.
8.	306	29	(7)	**Shannon, C.E.** A mathematical theory of communications. 3. Mathematical preliminaries. *Bell Syst. Tech. J.* 27:623-636, 1948.
9.	294	67	(67)	**Zygmund, A.** *Trigonometric series.* Cambridge, 1968.
10.	293	69	(25)	**Scheffe, H.** *Analysis of variance.* Wiley, 1959.
11.	291	66	(35)	**Courant, R. & Hilbert, D.** *Methods of mathematical physics.* Wiley, 1953-62.
12.	271	52	(34)	**Cramer, H.** *Mathematical methods of statistics.* Princeton, 1946.
13.	271	45	(45)	**Loeve, M.** *Probability theory.* 3rd ed. VNR, 1963.
14.	265	73	(31)	**Erdelyi, A.,** ed. *Tables of integral transforms.* 2 vols. McGraw, 1954.
15.	246	57	(30)	**Anderson, T.W.** *Introduction to multivariate statistical analysis.* Wiley, 1958.
16.	242	61	(61)	**Kelley, J.L.** *General topology.* VNR, 1955.
17.	235	57	(10)	**Kendall, M.G. & Stuart, A.** *Advanced theory of statistics.* 3 vols. Hafner, 1967-9.
18.	215	60	(45)	**Coddington, E.A. & Levinson, N.** *Theory of ordinary differential equations.* McGraw, 1955.
19.	211	62	(62)	**Zariski, O. & Samuel, P.** *Commutative algebra.* VNR, 1958.
20.	209	55	(55)	**Cartan, H. & Eilenberg, S.** *Homological algebra* (Mathematical series, vol. 19). Princeton, 1956.
21.	208	62	(51)	**Kato, T.** *Perturbation theory for linear operators.* Springer, 1966.
22.	197	36	(11)	**Pontryagin, L.S.** *et al. Mathematical theory of optimal processes.* Wiley, 1962.
23.	195	32	(7)	**Powell, M.J.D.** An efficient method for finding the minimum of a function of several variables without calculating derivatives. *Computer J.* 7:155-162, 1964.
24.	192	82	(82)	**Bourbaki, N.** *Algèbre commutative.* AW, 1973.
25.	181	41	(10)	**Kruskal, J.B.** Multi-dimensional scaling by optimizing goodness of fit to a nonmetric hypothesis. *Psychometrika.* 29:1-27, 1964.

A List of Heavily Cited Works in Mathematics, 1961-1972. This list shows, in order of decreasing frequency of citation, works in mathematics highly cited during the period 1961-1972. Column 1 shows the rank of item on this list. Column 2 shows the total number of times the item was cited by journals indexed in the *Science Citation Index* during the period 1961-1972. Column 3 shows the number of times the item was cited in 1972 by all *SCI* journals. Column 4 shows the number of times cited by *SCI* mathematics and statistics journals in 1972. Column 5 gives full bibliographic data. Publisher information in the case of

Rank	Total Citations 1961-1972	1972	Selective 1972	Bibliographical Data
(1)	(2)	(3)	(4)	(5)
26.	171	65	(23)	**Wilkinson, J.H.** *Algebraic eigenvalue problem.* Oxford, 1965.
27.	171	28	(28)	**Bass, H.** Finitistic dimension and a homological generalization of primary rings. *T. Amer. Math. Soc.* 95:466-488, 1960.
28.	166	55	(53)	**Gillman, L. & Jerison, M.** *Rings of continuous functions.* VNR, 1960.
29.	164	31	(9)	**Rosenbrock, H.H.** An automatic method for finding the greatest or least value of a function. *Computer J.* 3:175-184, 1960.
30.	164	25	(4)	**Scheffe, H.** A method for judging all contrasts in the analysis of variance. *Biometrika.* 40:87-104, 1953.
31.	162	18	(7)	**Bargmann, V.** Irreducible unitary representations of the Lorentz group. *Ann. Math.* 48:568-640, 1947.
32.	160	47	(25)	**Varga, R.S.** *Matrix iterative analysis.* PH, 1962.
33.	159	37	(37)	**Halmos, P.R.** *Measure theory.* VNR, 1950.
34.	158	26	(5)	**Shepard, R.N.** Analysis of proximities: multi-dimensional scaling with an unknown distance function. 1. *Psychometrika.* 27:125-140, 1962.
35.	157	32	(5)	**Muskhelishvilli, N.I.** *Singular integral equations.* S-H Serv., 1961.
36.	154	53	(50)	**Rao, C.R.** *Linear statistical inference and its applications.* Wiley, 1965.
37.	145	52	(51)	**Hartman, P.** *Ordinary differential equations.* Wiley, 1964.
38.	145	45	(34)	**Spanier, R.H.** *Algebraic topology.* McGraw, 1966.
39.	140	30	(10)	**Peterson, W.W. & Weldon, E.J., Jr.** *Error-correcting codes.* MIT, 1971.
40.	140	21	(4)	**Feynman, R.P.** Space-time approach to non-relativistic quantum mechanics. *Rev. Mod. Phys.* 20:367-387, 1948.
41.	139	23	(23)	**Gabriel, P.** Des Catégories abeliènnes. *B. Soc. Math. Fr.* 90:323-448, 1962.
42.	139	14	(14)	**Kervaire, M.A.** Groups of homotopy spheres. *Ann. Math.* 77:504-557, 1963.
43.	138	10	(5)	**Toller, M.** Three-dimensional Lorentz group and harmonic analysis of the scattering amplitude. *Nuovo Cimento.* 37:631-657, 1965.
44.	135	49	(44)	**Hewitt, E. & Ross, K.A.** *Abstract harmonic analysis. Vol. 1. Structure of topological group.* Springer, 1963.
45.	132	36	(12)	**Gelfand, I.M., Shilov, G.E., Vilenkin, N.Y. & Graev, M.I.** *Generalized Functions.* 5 vols. Academic, 1964.
46.	129	30	(4)	**Nelder, J.A. & Mead, R.** Simplex method for function minimization. *Computer J.* 7:308-313, 1965.
47.	129	26	(9)	**Kruskal, J.B.** Nonmetric multi-dimensional scaling: a numerical method. *Psychometrika.* 29:115-129, 1964.
48.	127	21	(5)	**Shepard, R.N.** Analysis of proximities: multi-dimensional scaling with an unknown distance function. 2. *Psychometrika.* 27:219-246, 1962.
49.	122	42	(28)	**Yoshida, K.** *Functional analysis.* Springer, 1965.
50.	120	53	(53)	**Gorenstein, D.** *Finite groups.* HR, 1968.

books has been abbreviated. Full information is as follows (as a matter of interest we have included in parentheses after the listed abbreviation the number of items each publisher contributed to the list): Academic (5) New York: Academic Press,Inc. Allyn (2) Rockleigh, N.J.: Allyn & Bacon, Inc. AMS (3) New York: AMS Press, Inc. AW (2) Reading: Addison-Wesley Publishing Co. Blaisdell (1) New York: Blaisdell Publishing Co., Inc. Cambridge (1) New York: Cambridge University Press. Gauthier-Villars (2) Paris: Gauthier-Villars. Hafner (1) New York: Hafner Publishing Co., Inc. HR (1) New York: Harper & Rowe

Rank	Total Citations 1961-1972	1972	Selective 1972	Bibliographical Data
(1)	(2)	(3)	(4)	(5)
51.	119	42	(42)	Rudin, W. *Fourier analysis on groups.* Wiley, 1962.
52.	117	46	(46)	Schaefer, H.H. *Topological vector spaces.* 3rd ed. (Graduate texts in mathematics series, Vol. 3). Springer, 1966.
53.	116	31	(6)	Torgerson, W.S. *Theory and method of scaling.* Wiley, 1958.
54.	115	37	(37)	Hall, M. *Theory of groups.* Macmillan, 1961.
55.	112	32	(28)	Helgason, S. *Differential geometry and symmetric spaces.* (Pure and applied mathematics series, Vol. 12). Academic, 1962.
56.	112	14	(14)	Feit, W. & Thompson, J.G. Solvability of groups of odd order. *Pacific J. Math.* 13:775-1029, 1963.
57.	111	53	(53)	Clifford, A.H. & Preston, G.B. *Algebraic theory of semigroups.* 2 vols., AMS, 1961.
58.	111	45	(45)	Nagata, M. *Local rings.* Wiley, 1962.
59.	110	31	(28)	Hoffman, K. *Banach spaces of analytic functions.* PH, 1962.
60.	109	28	(9)	Courant, R. & Friedrichs, K.O. *Supersonic flow and shock waves.* Wiley, 1948.
61.	107	40	(40)	Lambek, J. *Lectures on rings and modules.* Blaisdell, 1966.
62.	107	20	(19)	Harris, T.E. *Theory of branching processes.* PH, 1964.
63.	107	11	(5)	Toller, M. An expansion of the scattering amplitude at vanishing 4-momentum transfer using the representation of the Lorentz group. *Nuovo Cimento A.* 53:671-716, 1968.
64.	105	29	(7)	Fletcher, R. & Reeves, C.M. Function minimization by conjugate gradients. *Computer J.* 7:149-154, 1964.
65.	104	34	(24)	Mitchell, B. *Theory of categories.* Academic, 1965.
66.	102	11	(11)	Agmon, S. Estimates near the boundary for solutions of elliptic partial differential equations satisfying genery boundary conditions. *Comm. Pure App. Math.* 12:623-727, 1959.
67.	101	36	(36)	Rickart, C.E. *General theory of Banach algebras.* VNR, 1960.
68.	98	53	(34)	Harary, F. *Graph Theory.* A-W, 1969.
69.	98	37	(37)	Halmos, P.R. *Hilbert space problem books.* VNR, 1967.
70.	97	37	(37)	Billingsley, P. *Convergence of probability measures.* Wiley, 1968.
71.	95	26	(26)	Dugundji, J. *Topology.* Allyn, 1964.
72.	95	14	(14)	MacKey, G.W. Induced representations of locally compact groups. I. *Ann. Math.* 55:101-139, 1952.
73.	94	23	(23)	Friedman, A. *Partial differential equations of parabolic type.* PH, 1964.
74.	93	41	(34)	Huppert, B. *Endliche Gruppen.* Springer, 1967.
75.	93	19	(17)	Gunning, R.C. & Rossi, H. *Analytic functions of several complex variables.* PH, 1965.
76.	89	48	(48)	Dixmier, J. *Les algèbres d'operateurs dans l'espace Hilbertien.* Gauthier-Villars, 1957.
77.	86	22	(22)	Dieudonné, J. *Foundations of modern analysis.* Academic, 1969.
78.	82	17	(15)	Dixmier, J. *Les C* algèbres et leurs representations.* Gauthier-Villars, 1964.

Publishers, Inc. Macmillan (1) New York: Macmillan Company. McGraw (4) New York: McGraw-Hill Book Company. MIT (1) Cambridge, Mass.: M.I.T. Press. Oxford (1) New York: Oxford University Press. PH (5) Englewood Cliffs, N.J.: Prentice-Hall, Inc. Princeton (2) Princeton, N.J.: Princeton Univ. Press. S-H. Serv. (1) Riverside, N.J.: S-H. Service Agency, Inc. Springer (5) New York: Springer-Verlag, Inc. VNR (7) New York: Van Nostrand Reinhold Co. Wiley (14) New York: John Wiley & Sons, Inc.