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This Week's Citation Classic

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This letter describes a Monte Carlo computer algorithm meant to simulate the growth of an object by the accretion of a diffusing substance. In the algorithm a random walker moves on a lattice until it encounters a seed-cluster of previously occupied lattice sites. The walker then becomes part of this cluster at the point of contact. This process, when repeated, makes a fractal object whose fractal dimension *D* is about 1.7 in two dimensions. [The *SCI*® indicates that this paper has been cited in more than 1,130 publications.]

Fractal Growth: A Continuing Mystery

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In 1978 a graduate student named Steve Forrest showed me an intriguing electron micrograph and asked, "Do you think this is a fractal?" In those days the notion that the mathematical scale invariance known as fractal symmetry might occur in natural phenomena was just being popularized by Benoit Mandelbrot.[1] Steve's query came as a response to a talk I gave at the University of Michigan introducing fractals. His micrograph, made as part of his PhD research, showed aggregated colloidal iron of the kind used in magnetic recording. The intricate, wispy branches of Steve's aggregates *did seem* to have the hallmark feature of geometrically constructed fractals: The structure of the whole was reflected in each part We decided to look for quantitative evidence of fractal structures in the aggregates. Several months later we had amassed suggestive, but far from compelling, evidence. The main missing ingredient was a model: We had no idea why these colloidal iron particles might form a fractal structure.

We felt this weakness in our picture acutely as we discussed the aggregates with my colleague Len Sander. In an effort to make *some* link between the intricate structure and its origins, I worked up several order-of-magnitude estimates to give a quantitative idea of how the aggregates formed. These yielded a surprise: The aggregating particles seemed to arrive in the aggregation region one at a time and seemed to have no further motion than thermal Brownian motion when they stuck together. Could these two facts alone explain the fractal structure? It seemed impossible, yet a similar phenomenon happened in a different domain. Dendritic crystals growing in a super-cooled liquid also develop an increasingly subdivided structure—like that of snowflakes. This structure was also known to be controlled by diffusion, like the Brownian motion of our iron particles.

Len and I immediately saw that if these two elements were the essential ones to explain fractal growth, it would be very simple to devise a test. The two elements could easily be simulated in a computer by allowing a cluster of connected particles to grow by the successive addition of randomly moving particles. We were half sure that when we implemented this program the resulting structures would be mere amorphous blobs of particles. But to our delight the structure that emerged from Len's program was the intricate branched pattern now known as diffusion-limited aggregation (DLA).

DLA is now recognized as a widespread, important physical process. In phenomena as diverse as electrodeposition or the invasion of air into liquid-soaked rock, the quantitative fractal properties of the DLA computer model have been reported.[2] Generalizations of OLA have been devised to explain the electric breakdown of a highly charged insulator, and for the branched structure of trees, blood vessels, root systems, etc.[3,4] Except for the random walk, DLA is perhaps the simplest process that makes a fractal structure. As such, it serves as a paradigm for more complicated processes like turbulence or avalanches, where fractal-like structures are believed to occur.[3,4] But despite this simplicity, the origins of the fractal structure of DLA remain a mystery. To explain this symmetry mathematically is one of the fundamental unsolved problems of statistical physics.5

For all its ramifications, DLA turned out not to account for the aggregation phenomenon that inspired it. This aggregation was soon found[6,7] to be a separate process, with widespread implications of its own.[8]

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