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Alexander S & Orbach R. Density of states on fractals: "fractons." *J. Phys.—Lett.* 43:L625-31, 1982.

[Racah Institute, Hebrew University, Jerusalem, Israel; and, Ecole Supérieure Physique Chimie Industrielles Ville Paris, France]

A new scaling dimension, d , determines the density of states and dispersion on a fractal geometry. For percolation clusters $d = 4/3$. The derivation uses the formal connection of the density of states to the probability distribution for diffusion. [The SC® indicates that this paper has been cited in more than 865 publications.]

count states in q -space and one was, obviously, tempted to try to generalize this to fractals.

I had, however, worked very hard on such a mapping,¹ mainly that of the Landau-Ginzburg equations.² I even had a prescription for calculating the density of states by explicit mode counting which was as yet unpublished. (I was about halfway through my struggle with *Physical Review B*.) I therefore knew that the density of states had to depend on the internal geometry of the fractal. One found different densities of states for different fractals with the same D . It followed that a new nontrivial index, or dimension d , was involved. This also meant that counting states in q -space was not very promising.

Dynamics on Self-Similar Fractals

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One needed a different procedure and such a procedure was indeed available. We had studied anomalous diffusion³ and had used the general scaling relationship between the probability distribution for diffusion and the density of states.⁴ I also knew that diffusion on fractals was anomalous and even how the index was related to the conductivity index.⁵ This amounted to having all the pieces at hand so that one could write down the expressions for the dispersion and for the density of states.

I usually have to work quite hard on my papers and readers also tend to find them hard to read. Occasionally one does, however, run into a problem where things just fall into place. The answers seem obvious, without effort, and you have a simple and intuitive derivation for the results. This usually happens when, because of what you were doing previously, you are looking at the problem from a different angle. If you are very lucky you provide answers for questions of real interest. You may then produce a *Citation Classic*®. I do not claim that this is the only way but, for a theoretical physicist, it is certainly a way.

For aesthetic and didactic reasons it seemed useful to work out some nontrivial examples. Using Stauffer's simulation values⁶ for the percolation D and conductivity index, t , in our expression for β resulted in the very surprising $\beta = 4/3$ conjecture. This was certainly important in making the paper popular. The first signs of the "4/3" excitement are already in Tom Lubensky's appendix to our paper.

The present paper is a case in point.

Ray Orbach was in Paris. He wrote to me asking whether I believed that the fractal dimension, D , describes the density of states on fractals as the dimension usually does. Some friends of his were claiming this. The answer was obvious.

In writing this commentary I reread our 1982 letter and liked it. I think that, except for the neglect of rotational invariance, it still contains almost all that one knows about dynamical scaling on complex geometries—and no errors. It may however not be as simple to read as I remembered and some of the more subtle points were certainly overlooked by many readers at the time. Numerically "4/3" is a much better approximation but if this really has some deeper meaning it is still unknown.

What one needed was to map a laplacian, or the continuum limit of a second order difference equation, on a fractal geometry and then to calculate the density of states. The standard textbook procedure is to use the dispersion to

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 3. Alexander S, Bemasconi J, Schneider W R & Orbach R. Excitation dynamics in random 1-dimensional systems. *Rev. Mod. Phys.* 53:175-98, 1981. (Cited 380 times.)
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