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Kosterlitz J M & Thouless D J. Ordering, metastability and phase transitions in two-dimensional systems. *J. Phys.—C—Solid State Phys.* 6:1181-203, 1973.
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This paper showed how a type of order more robust than the conventional sort of long-range order can exist in two-dimensional systems. When the temperature is raised, the ordered state is destroyed by the dissociation of pairs of defects such as vortices or dislocations, and a new type of phase transition is produced. [The *SCI*® indicates that this paper has been cited in more than 1,930 publications, making it the most-cited paper in this journal.]

Defect-Driven Phase Transitions

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Arguments against the existence in two-dimensional systems of the type of long-range order associated with superfluidity in helium, freezing of solids, or magnetization of isotropic magnets had been given in the 1930s by R.E. Peierls and L.D. Landau, and had been made rigorous in the 1960s by P.C. Hohenberg, and by N.D. Mermin and H. Wagner. Despite this, there were many theoretical and experimental indications that some sort of ordered state existed at low temperatures for two-dimensional systems such as helium films.

The idea that superfluidity might be destroyed by the spontaneous formation of quantized vortices, or solids melted by the formation of dislocations, is an old one, but it never looked convincing for bulk systems. In two dimensions, the statistical mechanics of such defects is particularly simple, as the position of a defect is given by a point, whereas it is specified by a path in three dimensions. We used the close analogy between the behavior of a collection of defects and that of a two-dimensional gas of positive and negative electric charges. At low temperatures, all the charges are bound in pairs of "molecules," and at some critical temperature, the largest of these molecules dissociated to form a conducting plasma. This is the nature of the phase transition that we argued should exist in superfluid films, in two-dimensional solids, and in two-dimensional magnets with a preferred plane of magnetization.

I stumbled across this idea while preparing a course of lectures on superfluidity and superconductivity, but I was prepared to expect the peculiar nature of the phase transition by work I had done on a one-dimensional problem two years earlier. I recruited J. Mike Kosterlitz, who was then a postdoctoral particle theorist and is now at Brown University, to work on this problem. This paper was the result of our collaboration. Before we had published it, we became aware that Berezinskii,[1] in the Soviet Union, had anticipated some of our ideas, although our use of the renormalization group[2] gave us insights that he missed.

We got many of the interesting features of the transition right, but there were important modifications that were made by other people. It was pointed out that, in addition to the spontaneous formation of dislocations in a solid, associated with the start of viscous flow, there was a further transition of the same sort possible in which disclinations would appear, and orientational order lost. We had argued that this sort of transition should not occur for superconductivity but had forgotten that the penetration depth in a thin film is very large.

The initial reaction of the physics community to this work was polite interest rather than enthusiasm, and it was several years before an experimental test was made. In 1978, D.J. Bishop and J.D. Reppy[3] published the results of their experiment on the superfluid density of a helium film, and correlated their results with other experiments to show the constancy of the ratio of superfluid density to transition temperature; this is a characteristic of the theory which was convincingly demonstrated by D.R. Nelson and Kosterlitz[4] in 1977. The dramatic experimental results started the boom in work on this theory, and on its experimental manifestations. Two recent reviews of the theory and its applications have been written by P. Minnhagen[5] and K.J. Strandburg.[6] The layered copper oxide superconductors have helped to maintain activity in this subject.

1. **Berezinskii V L.** Destruction of long-range order in one-dimensional and two-dimensional systems possessing a continuous symmetry group. II. Quantum systems. *Zh. Eksp. Teor. Fiz. SSSR* 61:1144-56, 1971. (Cited 110 times.)
2. **Kosterlitz J M.** The critical properties of the two-dimensional *xy* model. *J. Phys.—C—Solid State Phys.* 7:1046-60, 1974. (Cited 915 times.)
3. **Bishop D J & Reppy J D.** Study of the superfluid transition in two-dimensional ^{4}He films. *Phys. Rev. Lett.* 40:1527-30, 1978. (Cited 175 times.)
4. **Nelson D R & Kosterlitz J M.** Universal jump in the superfluid density of two-dimensional superfluids. *Phys. Rev. Lett.* 39:1201-5, 1977. (Cited 285 times.)
5. **Minnhagen P.** The two-dimensional Coulomb gas, vortex unbinding and superfluid-superconducting films. *Rev. Mod. Phys.* 59:1001-66, 1987.
6. **Strandburg K J.** Two-dimensional melting. *Rev. Mod. Phys.* 60:161-307, 1988.

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