CC/NUMBER 45
NOVEMBER 5, 1990

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Bradshaw P. The analogy between streamline curvature and buoyancy in turbulent shear flow. *J. Fluid Mech.* 36:177-91, 1969.
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The general improvement in calculation methods for turbulent boundary layers in the late 1960s made it clear that longitudinal surface curvature ("camber") had a significant effect on the flow. It turned out that the analogy between the effects of "centrifugal force" and of buoyancy force had been suggested in qualitative form by Prandtl in 1930! [The *SCI*® indicates that this paper has been cited in more than 115 publications.]

Effect of Streamline Curvature on Turbulence

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August 13, 1990

In 1965 Brian Thompson,[1] working with my late teacher Mac Head at Cambridge, produced a purely empirical correlation effect for the affect of "centrifugal force" on turbulence, which set me thinking about the underlying physics. To quote the 1969 paper, "It is well known that certain laminar flows over curved or rotating boundaries have analogues in flows with buoyancy." The Richardson number, Ri, a well-known meteorological parameter, has a fairly obvious analog in curved flows if one substitutes centrifugal force for buoyancy force. It seemed probable that the analogy for *turbulent* flow should be at least qualitatively useful, and it was fun to go through the various meteorological parameters and derive their curved-flow analogs.

The primary reference of my 1969 paper was to Prandtl's paper of 1930,[2] with a unified discussion of buoyancy and curvature effects well in advance of useful experimental data. I'm embarrassed to say that I had not read Prandtl's German paper until my old friend George Mellor of Princeton University pointed it out to me.

Prandtl assumed that the fractional change in mixing length, say, should be of the same order as the dimensionless parameters that correlate buoyancy or curvature effects. However, meteorological data showed a percentage change in mixing length that was an order of magnitude greater than Ri, because buoyancy changes eddy processes rather spectacularly—as any cloud-watcher will confirm. Prandtl was a great man, but he occasionally missed the subtleties of the phenomenon he was studying. For example, his "mixing-length" concept of lumps of fluid (*Flüssigkeitsballen*) exchanging momentum may[3] have been inspired by the Ahlborn method of flow visualization using particles scattered on a liquid surface: unfortunately, the two-dimensionality enforced by the flat surface suppresses the essential mechanisms of turbulence.

My 1969 paper, later expanded,[4] suggested correlating the affects of streamline curvature on the dimensionless parameter (U/R)/(dU/dy), a first-order analog of the Ri number. The results are in semiquantitative agreement with meteorological work, but the mechanisms of "destabilizing" and "stabilizing" buoyancy or curvature are qualitatively different, and these differences are reflected in the behavior of curved flows.[5]

The large effects of streamline curvature were surprising at the time: Today it is regretfully accepted that various kinds of perturbation of a simple shear layer can produce disproportionately large changes in turbulence structure.

1. **Thompson B G J.** The calculation of shape-factor development in incompressible turbulent boundary layers with or without transpiration. *AGARDograph* 1965:97. (Cited 5 times.)
2. **Prandtl L.** *Influence of stablizing forces on turbulence* (1930). Reprinted in: **Tollmien W, Schlichting H & Görtler H.** *Ludwig Prandtl. Gesammelte Abhandlungen.* Berlin, FRG: Springer-Verlag, 1961. Vol. 2. p. 778.
3. **Bradshaw P.** Possible origin of Prandtl's mixing-length theory. *Nature* 249:135-6, 1974.
4. **Bradshaw P.** Effects of streamline curvature on turbulent flow. *AGARDograph* 1973:169. (Cited 160 times.)
5. **Barlow R S & Johnston J P.** Structure of a turbulent boundary layer on a concave surface. *J. Fluid Mech.* 191:137-76, 1988. (Cited 5 times.)

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