

# This Week's Citation Classic®

**Shepard R. N.** Attention and the metric structure of the stimulus space.  
*J. Math. Psychol.* 1:54-87, 1964.  
(Bell Telephone Laboratories, Murray Hill, NJ)

For stimuli (like colors) that are not perceptually analyzed into separate features, perceived similarities have conformed with a stable Euclidean metric. For stimuli that are analyzable into separate features of size and orientation, similarities are here shown to favor the city-block metric and to depend on the subject's state of attention. [The *SCI*® and the *SSCI*® indicate that this paper has been cited in over 215 publications.]

## Similarity Metrics for Psychological Space

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As a Stanford University undergraduate, I was torn between the formal sciences (mathematics, physics) and the mental sciences (philosophy, psychology). It seemed that never the twain would meet. Subjective experience, though the source of each individual's objective and quantitative knowledge, did not itself appear susceptible to objective study or to quantitative formalization.

Then, in 1950, I learned of a doctoral dissertation that Fred Attneave had just completed in psychology at Stanford. Attneave showed that subjective ratings of the perceived dissimilarities between stimuli and objective frequencies of errors while learning to identify those stimuli could both be explained in terms of distances between stimuli in a "psychological space."<sup>1</sup> It was this idea of representing subjective phenomena geometrically that inspired me to pursue a career in psychological science and, a decade later, to devise the first "nonmetric" method of multidimensional scaling (also a *Citation Classic*).<sup>2</sup> For 40 years this idea has continued to guide my research.<sup>3,4</sup>

Curiously, the psychological distances that seemed best to explain Attneave's data were non-Euclidean ("city-block") distances, com-

puted as sums of differences between stimuli on underlying spatial dimensions, rather than Euclidean distance, computed as the square roots of sums of squares of those differences. Attneave's stimuli were perceptually analyzable, however, into separate dimensions of visual lightness, size, and shape. In ensuing work with colors differing along perceptually less separable dimensions of lightness and saturation, W.S. Torgerson, in a 1951 Princeton University dissertation using subjective judgments of similarity (see reference 5), and I, in a 1955 Yale University dissertation using objective errors during identification learning, obtained good fits with the Euclidean metric.<sup>4</sup> Did the metric of psychological space depend, then, on the perceptual analyzability of the stimuli?

The experiments reported in my 1964 paper were the first specifically designed to address this question. I chose stimuli differing in size and orientation in a way that would sharply discriminate between the Euclidean and city-block metrics. The subjective judgments and objective error frequencies I obtained both favored the city-block over the Euclidean metric for these stimuli. Together with results on classification learning that my coworkers (C.I. Hovland and H.M. Jenkins in 1961 and J.-J. Chang in 1963) and I had recently reported, these new findings also demonstrated a fundamental role of selective attention for analyzable stimuli.<sup>4</sup>

Subsequent studies using similarity judgments (by R. Hyman, A. Well, and others) and classification performances (by W.R. Garner, G.R. Lockhead, and others) have confirmed and elaborated the distinction between unitary stimuli differing on perceptually "integral" dimensions and analyzable stimuli differing on perceptually "separable" dimensions.<sup>4,6</sup> In my most recent effort toward a formal account of the mental, I discovered that all these empirical results are mathematically entailed by a theory of generalization based on fundamental principles of the world in which we have evolved.<sup>3,4</sup>

1. Attneave F. Dimensions of similarity. *Amer. J. Psychol.* 63:516-56, 1950. (Cited 170 times.)
2. Shepard R N. The analysis of proximities: multidimensional scaling with an unknown distance function (I & II). *Psychometrika* 27:125-40; 219-46, 1962. (Cited 585 and 555 times, respectively.) [See also: Shepard R N. *Citation Classic*. (Smelser N J, comp.) *Contemporary classics in the social and behavioral sciences*. Philadelphia: ISI Press, 1987. p. 6.]
3. . . . . . Toward a universal law of generalization for psychological science. *Science* 237:1317-23, 1987. (Cited 15 times.)
4. . . . . . Integrality versus separability of stimulus dimensions: from an early convergence of evidence to a proposed theoretical basis. (Lockhead G R & Pomerantz J R, eds.) *Perception of structure*. American Psychological Association. (In press.)
5. Torgerson W S. Scaling and test theory. *Annu. Rev. Psychol.* 12:51-70, 1961. (Cited 1,425 times.)
6. Garner W R. *The processing of information and structure*. Potomac, MD: Erlbaum, 1974. 203 p. (Cited 570 times.)