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This Week's Citation Classic®

Atiyah M F & Singer I M. The index of elliptic operators: I. *Ann. Math.* 87:484-530, 1968. **Atiyah M F & Segal G B.** The index of elliptic operators: II. *Ann. Math.* 87:531-45, 1968. **Atiyah M F & Singer I M.** The index of elliptic operators: III & IV. *Ann. Math.* 87:546-604, 1968; 93:119-49, 1971. [Mathematical Inst., Univ. Oxford, England; Inst. for Advanced Study, Princeton, NJ; Massachusetts Inst. Technology, Cambridge, MA; and St. Catherine's Coll., Univ. Oxford, England]

These papers give a general formula for the index of an elliptic differential operator on a compact manifold. When there is a compact symmetry group, a corresponding character formula is established. The proofs depend on an extensive use of K-theory. [The *SCI*® indicates that these papers have been cited a total of 655 times.]

Mathematics and Physics—A Convergence

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It is now 25 years since I.M. Singer and I began our investigation into the index of elliptic operators. For more than a decade, this was the focal point of our joint research, and it developed in many different directions. Because the index theorem plays an important role in a variety of areas and links together many branches of mathematics, including analysis, geometry, and topology, it has always attracted wide attention. However, in the past decade it has also turned out to be important in theoretical physics in connection with gauge theories. It is undoubtedly this aspect that has led to its prominence in the citation list.

Singer and I began our collaboration when he spent a sabbatical term with me in Oxford. I was at that time trying to understand some formulas of F. Hirzebruch related to spin manifolds, and growing out of his work on the Riemann-Roch theorem in algebraic geometry.[1] Hirzebruch's work suggested that there might be a formula involving the dimension of the "harmonic spinors" on a manifold. This led us to rediscover the Dirac equation in the context of Riemannian geometry and to formulate the index theorem for the Dirac operator.

Shortly afterwards Stephen Smale passed through Oxford and told us that I.M. Gel'fand[2] had posed the general problem of computing, by topological methods, the index of a general elliptic operator. This turned our attention to the general index problem and, based on the key example of the Dirac operator, we were quickly led to formulate the correct general formula.

The proof took much longer. In fact, over the succeeding years, many different proofs have emerged, each with its own merits.

When Singer and I first stumbled on the Dirac operator, we were very far from making any connection with physics. We were amused and slightly intrigued that our geometrical investigations should have some analogy with Dirac's work, but we certainly had no idea that our index theorem would subsequently be given a physical interpretation. It took many years before mathematicians and physicists realized that their interests were converging and that the index of the Dirac operator was of central importance to both sides.

Paper 2 of the series, written jointly with G.B. Segal, is based on K-theory techniques, and the development of K-theory went hand-in-glove with the progress of index theory. This aspect has blossomed in recent years with the emphasis on the applications to noncommutative C^*-algebras and noncommutative geometry in the sense of A. Connes.[3]

1. **Hirzebruch F.** *Topological methods in algebraic geometry.* New York: Springer-Verlag, 1966. (Cited 315 times.)
2. **Gel'fand I M.** On elliptic equations. *Russ. Math. Survey.—Engl. Tr.* 15:113-23, 1960. (Cited 10 times.)
3. **Connes A.** Non-commutative differential geometry. *Publ. Math.—Paris* (62):257-360, 1985. (Cited 5 times.)

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