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Anderson P W, Halperin B I & Varma C M. Anomalous low-temperature thermal properties of glasses and spin glasses. *Phil. Mag.* 25:1-9, 1972.  
[Bell Laboratories, Murray Hill, NJ]

We show that a linear specific heat at low temperatures for glass follows naturally from general considerations on the glassy state. From the same considerations we obtain the experimentally observed anomalous low-temperature thermal conductivity, and we predict an ultrasonic attenuation that increases at low temperatures. [The *SCI*® indicates that this paper has been cited in over 1,060 publications.]

## Quantum Mechanics in Glasses

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In the summer of 1971, on a Friday, Professor Robert O. Pohl of Cornell University gave a seminar at Bell Laboratories on his measurements of the specific heat and thermal conductivity of a variety of glasses.<sup>1</sup> At low temperatures the specific heat was always linear and the thermal conductivity, quadratic in temperature. Even more remarkably the coefficients were within a factor of about two, the same irrespective of the variety of glass.

I had just been given a permanent job at Bell Labs after two years as a postdoc in the theoretical physics group. I had spent the summer working with a colleague, V. Narayanaswami, on nonlinear (soliton) propagation of heat in solids. Narayanaswami had told me about his earlier work on tunneling states in solids—like those of  $\text{OH}^-$  in alkali halides. I knew that the specific heat of such defects is proportional to  $(\Delta/T)$ , where  $\Delta$  is the tunnel splitting. So it was obvious that, if one had a reasonably uniform distribution of  $\Delta$ 's, a linear specific heat would result.

This line of thinking seemed to fit well with one's preconception that glasses should have zero-point entropy and metastable distribution of atomic configurations, arising due to competing interactions and strong anharmonicity. After cogitating on this over the weekend, on Monday afternoon I went to the office of our most distinguished theoretical physicist, Phil Anderson. I drew a double-well potential and told him my speculations. He had been thinking along similar lines, starting from zero-point entropy of glasses. He recalls that, during Pohl's talk, a picture grew in his mind of connections between these configurations so that they become accessible to each other over finite barriers distributed randomly in height. The scale of variation of the barriers as well as of the energy difference would be the melting

temperature of glass. This provides the right sort of magnitude of the linear specific heat.

Anderson asked me about the thermal conductivity. On Tuesday I tried to calculate the phonon mean free paths to get the thermal conductivity but got nowhere. Anderson stayed home on Tuesday. On Wednesday morning, I met him in our seminar room and he handed me a manuscript that had the theory not only of the specific heat and the thermal conductivity but also an application of the ideas to spin glasses. The next week we learned that Bert Halperin in our group at Bell Labs was thinking along similar lines. Bert added the section on ultrasonic attenuation and rewrote the paper in his characteristically careful and polished fashion.

We submitted the paper to *Philosophical Magazine* rather than *Physical Review Letters* because in those days Anderson was disgruntled with the latter's editorial and refereeing processes. Soon thereafter we learned that W.A. Phillips,<sup>2</sup> a postdoc at Stanford, had written a paper with similar ideas.

These papers became very popular indeed. They were easy to understand, and they found resonance with everybody's conception of the glassy and disordered state, on which work began in earnest in the 1970s and is continuing. They also made specific predictions for a variety of experiments. Especially dramatic were experiments on the time-dependence of the specific heat, saturation of ultrasound, and various two-level phenomena like phonon echoes.

The phenomenological theory was qualitatively confirmed in all the experiments, but close examination revealed that a parameter obtained by fitting to one kind of experiment had to be doctored, often by a factor of 2-4 to get quantitative agreement with another kind of experiment. No really cogent explanation of such discrepancies has yet emerged, nor any microscopic understanding of the tunneling states nor of the almost universal values for the coefficients of the linear specific heat or of the quadratic thermal conductivity. A good critique of the state of affairs has recently been written;<sup>3</sup> it suggests an explanation not relying on tunneling states, but it is not free from difficulties.

The problem is very hard. Considerable progress has been made on a related but simpler problem: that of spin glass where the concept of a new type of order (in time) has been introduced, again by Anderson, together with S.F. Edwards. In spin glasses, the disorder is given by the competing interactions, which are frozen. In glasses, the interactions adjust consistently to the state they give rise to. In spin glasses, the concept of frustration has played a dominant role. Perhaps to understand glasses we need to introduce the concept of adjustment to frustration and to treat it mathematically. The renormalization of tunneling states due to anharmonicity (which is not by itself strong enough to produce bistability) is another way of saying the same thing.

1. Zeller R C & Pohl R O. Thermal conductivity and specific heat of noncrystalline solids. *Phys. Rev. B—Solid State* 4:2029-41, 1971. (Cited 520 times.)
2. Phillips W A. Tunneling states in amorphous solids. *J. Low Temp. Phys.* 7:351-60, 1972. (Cited 840 times.)
3. Yu C C & Leggett A J. Low temperature properties of amorphous materials: through a glass darkly. *Comments Condensed Matter Phys.* (In press.)

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