CC/NUMBER 47
NOVEMBER 24, 1986

This Week's Citation Classic®

Ljung L. Analysis of recursive stochastic algorithms.
IEEE Trans. Automat. Contr. **AC-22**:551-75, 1977.
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Recursive algorithms where random observations enter are studied in a fairly general framework. It is shown how problems like convergence with probability one and asymptotic behavior of the algorithm can be studied in terms of an associated ordinary differential equation. [The *SCI*® indicates that this paper has been cited in over 160 publications.]

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September 15, 1986

In 1970, as a fresh PhD student under K.J. Åström at the Automatic Control Department in Lund, Sweden, I started to make some simulations of stochastic, self-learning classification. I noted that the algorithms sometimes converged and sometimes diverged, and the mechanisms for this were not clear. It turned out that the key observation was what type of sign-change a certain function had around the expected convergence point. I set out to prove and generalize this observation.

A first report on the subject was written in 1972, before I left for Moscow to visit Ya.Z. Tsypkin under an Academy of Sciences exchange program. Many of the technical details of generalizing the proofs were then settled during long walks in Gorky Park. The generalizations led to the areas of stochastic approximation, adaptive control, and recursive identification. I felt that the latter two were starving for some analytical tools to investigate convergence.

Although the results were part of my PhD thesis in 1974, I did not write up the journal article version until 1976, when the results had been further streamlined and generalized. The reviewing procedure produced an unusual result—the reviewers asked for a *longer* version, and the published paper grew to 25 pages. It received the IEEE Control Systems Society's outstanding paper award in 1979.

The techniques of proving convergence have been fairly widely used in various applications. They form a basis for my book with T. Söderström.[1] The theorems have also been reproved using more sophisticated probabilistic tools.[2,3] Recently, I became aware of their relationship to so-called averaging in differential equations, which apparently is a "classical" subject. I judge my main contribution as realizing the underlying structure of many adaptive schemes and completing the quite technical proofs that the feedback mechanisms in these schemes necessitate.

Much of the success of the article I attribute to the more relaxed European publication attitude. Not being subject to the publish or perish peril, I enjoyed the luxury of spending five years "completing the research before writing the paper."

1. **Ljung L & Söderström T.** *Theory and practice of recursive identification.* Cambridge, MA: MIT Press, 1983. 529 p.
2. **Kushner H J & Clark D S.** *Stochastic approximation methods for constrained and unconstrained systems.* New York: Springer-Verlag, 1978. 261 p.
3. **Metivier M & Priouret P.** Applications of a Kushner and Clark lemma to general classes of stochastic algorithms. *IEEE Trans. Inform. Theory* **IT-30**:140-51, 1984.

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