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Drazin P G & Howard L N. Hydrodynamic stability of parallel flow of inviscid fluid. *Advan. Appl. Mech.* 9:1-89, 1966.

[Dept. Mathematics, Univ. Bristol, England and Dept. Mathematics, Massachusetts Inst. Technology, Cambridge, MA]

The theory of the linear instability of parallel flows of inviscid fluids is reviewed, with and without the effects of various force fields. The results are related to those on the propagation of stable waves, e.g., internal gravity waves, in these flows. Many results for particular basic flows are collected to illustrate the general theory. [The *SCI*® indicates that this paper has been cited in over 130 publications since 1966.]

P.G. Drazin
School of Mathematics
University of Bristol
Bristol BS8 1TW
England

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"L.N. Howard and I had common interests in hydrodynamic stability in 1958, when I joined the applied mathematicians at the Massachusetts Institute of Technology as a research associate of C.C. Lin. However, it was in the first Summer Study Program in Geophysical Fluid Dynamics at the Woods Hole Oceanographic Institution, in 1959, that Howard and I began to work together. We eventually wrote four papers[1-4] on the stability of parallel flows of inviscid fluids, in addition to various papers jointly with others and individually. Of the latter, Howard's ingenious and elegant note[5] on Richardson's criterion and the semicircle theorem has already appeared as a *Citation Classic*.[6] The present paper, being chiefly a survey as well as the last of our four papers, might also be regarded as their epitome.

"We wrote the paper long after my return to England in 1960. It was action at a distance. However, we met at various places several times before and during our collaboration to plan the work and sort out some of the fundamental ideas. Also we corresponded. One of us would draft a section and send it to the other. Then the other, perhaps after substantial further work, would redraft it. The process was iterated until it converged. The convergence was usually rapid.

"I can suggest three reasons our paper has been found useful. First, it is applicable in such fields as astrophysics, engineering, meteorology, and oceanography, as well as fluid dynamics.

"Secondly, the paper is a synthesis of diverse results by scores of authors. We emphasized the unity of the theory, demonstrating that similar physical ideas and mathematical methods are common to the theories of sound waves, internal gravity waves, Rossby waves, and magnetohydrodynamic waves, as well as the stability of shear flows. We included some new results for particular flows (and these have stimulated further work), but the more fundamental new parts of the paper (on nonparallel flows, on radiating modes with infinite over-reflection, and on physical mechanisms of instability) have not attracted much interest. Indeed, our work on radiating modes has been attributed to others after its rediscovery, perhaps because in 1966 we did not recognize its significance.

"Thirdly, the paper was timely. The fundamentals of the linear theory had been essentially worked out by 1966, so the paper has not dated rapidly. However, the stability characteristics of only a few flows had been computed then, and the paper may have stimulated some of the many subsequent computations. Similarly, the theory of nonlinear stability and waves in fluids was beginning to flourish. Again, the experimental and observational support of the theory was very poor but has blossomed since.

"A few books covering waves in, and stability of, shear flows have been written since 1966. The latest[7] cites some recent papers and books, yet neither it nor any other publication contains an attempt to update our paper comprehensively."

1. **Drazin P G & Howard L N.** Stability in a continuously stratified fluid. *Proc. Amer. Soc. Civil Eng. EM Div.* 87:101-16, 1961.
2. ----------. Stability in a continuously stratified fluid. *Trans. Amer. Soc. Civil Eng.* 128:849-64, 1963.
3. ----------. The instability to long waves of unbounded parallel inviscid flow. *J. Fluid Mech.* 14:257-83, 1962.
4. **Howard L N & Drazin P G.** On instability of parallel flow of inviscid fluid in a rotating system with variable Coriolis parameter. *J. Math. Phys.—NY* 43:83-99, 1964.
5. **Howard L N.** Note on a paper of John W. Miles. *J. Fluid Mech.* 10:509-12, 1961.
6. ----------. Citation Classic. Commentary on *J. Fluid Mech.* 10:509-12, 1961. *Current Contents/Engineering, Technology & Applied Sciences* 13(50):18, 13 December 1982.
7. **Drazin P G & Reid W H.** *Hydrodynamic stability.* Cambridge, England: Cambridge University Press, 1981. 525 p.