The article derives the consequences of the symmetries postulated by the special theory of relativity on the quantum mechanics of elementary particles and elementary sys--tems. It shows that the properties of such particles or systems are completely deter-mined by these symmetry postulates when given their mass and their angular momen--tum at rest. [The *SCI®* indicates that this paper has been cited over 430 times since 1961.]

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"The article in question, though of an entirely mathematical nature, was conceived to answer a question of physics, more particularly of quantum mechanics. Quantum mechanics' description of the state of a system is very different from those of classical theories, which give positions, veloci-ties, etc. of the constituents. Quantum mechanics describes the state of the system by a vector in an abstract space, called complex Hilbert space, a space of infinite dimensions. Naturally, some further prescriptions are then needed to obtain from the knowledge of the components of this vector the possible outcomes of observations on the system.

"Quantum mechanics' description of the state is 'linear' in the sense that if we have, for instance, two vectors which represent states with the same energy, the sum or any linear combina-tion of these vectors also represents states with the same energy. If ψ_1 and ψ_2 represent states with energy E, all the states $\alpha_1 \psi_1 + \alpha_2 \psi_2$ represent such states where the α are arbitrary com-plex numbers. This observation applies not only to states with definite energy but also to states in which some other physical property is specified. If this is the case, one can choose a set of states ψ_1, ψ_2, ψ_3,... in such a way that all states with this property (e.g., all states of energy E) can be written as linear-combinations $\alpha_1 \psi_1 + \alpha_2 \psi_2 + \cdots$ of these.

"If the specified set of states is invariant under some transformations—as are the states of an atom of definite energy under rotations, the state obtained from one of the ψ_i, under a rotation R, called $O_R \psi_i$, can be written as a linear-combination of the original states. The coefficients are denoted by $D(R)_{ki}$ so that $O_R \psi_i = \Sigma_k D(R)_{ki\psi k}$.

The $D(R)_{kj}$ satisfy some mathematical relations due to the fact that if the state $O_R \psi_i$ is subject to another transformation S one obtains, naturally, the state ψ_i rotated by SR, i.e., the state $O_{SR\psi i}$. The equations so resulting have been solved by the mathematicians Frobenius, Schur, and Weyl, and their usefulness for the theory of atomic spectra (the states of an atom of definite energy retain this property even if subject to a rotation) was recognized already in 1927.

"The present article extends this con-sideration to *all* states of a system — these are invariant under all transformations of the special theory of relativity (the inhomogeneous Lorentz group, that is Lorentz transfor-mations plus displacements in space and time). The article extends the results of the aforementioned mathematicians to this group —a group of infinite volume. For physics, the results proved most useful when ap-plied to elementary particles, but in mathematics, they started the exten-sion of the aforementioned results to infinite groups.

"It may be of some interest to recall that when the manuscript was submit-ted to one of our mathematical jour-nals, it was rejected as uninteresting.' However, when this writer mentioned this to John Von Neumann, one of the finest mathematicians of this century and one of the editors of the *Annals of Mathematics,* he said, 'We would be very pleased if you gave us this article for publication; we'd like to have it in the *Annals'* Not all articles originally rejected by a journal prove to be valueless."