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This Week's Citation Classic

Bankoff S G. A variable density single-fluid model for two-phase flow with particular reference to steam-water flow. *J. Heat Transfer* 82:265-72, 1960.
[Dept. Chemical Engineering, Northwestern Univ., Evanston, IL]

A model is presented for the equivalence, in terms of average void fraction and frictional pressure drop, of an intimately mixed gas-liquid flow in a conduit to the flow of a single fluid with position-dependent physical properties. [The *SCI*® indicates that this paper has been cited in over 130 publications since 1961, making it the 2nd most-cited paper published in this journal.]

S. George Bankoff
Chemical Engineering Department
Northwestern University
Evanston, IL 60201

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"This paper was written in the course of a summer appointment at Argonne National Laboratory, during a period of personal stress, and I certainly never thought that it would be regarded as a classic piece of work. It was at a time when boiling-water nuclear reactors were just being developed, and the science of two-phase flows and heat transfer was still in its infancy. Mechanistic models were needed to describe the fact that in an upward or horizontal steam-water flow the steam flows faster than the water. My model, taking into account radial distributional effects, was the simplest that could be derived which quantified the steam-water velocity ratio, and at the same time enabled the designer to predict the frictional pressure drop. A flow parameter, K, was introduced which was the ratio of the product of the averages of the gas volumetric concentration and the mixture velocity to the average of the products, and hence was related to a covariance. Because of its simplicity, it quickly found its way into texts and course notes. Empirical modifications were later introduced by several investigators which allowed the extension of the slip velocity ratio model to a variety of flow regimes. In these forms it has found its way into computer codes for light-water nuclear reactor safety calculations, oil-gas transport in pipelines, and other calculations.

"It is interesting, however, that shortly afterward, it led directly, by way of minor modifications, to a more famous model, due principally to Zuber,[1] called the 'drift flux model.' In this form, which also takes into account the local rise velocity of the bubbles relative to the surrounding liquid, it has been enshrined in the two-phase flow literature, and is today probably the most important single concept in two-phase flow modeling.

"The terms 'drift flux' and 'drift velocity' had been previously introduced by Graham Wallis,[2] who developed the continuity relationships and introduced the idea of continuity waves into two-phase flow. Thus, the idea of distributional effects turned out to be quite fruitful, and the drift velocity concept was extended to separated, as well as dispersed, flows. In the process, however, the scope was reduced, since the connection to frictional pressure drop was lost.

"Even more interesting is the fact that a doctoral student of mine, L.G. Neal,[3] made a similar extension of my model while a postdoctoral fellow in Norway, before the Zuber paper. However, his work appeared only in an obscure report from the nuclear research center at Kjeller, Norway, and has been since forgotten. I think that it is time that his contribution was recognized.

"The basic idea of cross-sectional area averaging of the mass and momentum equations has since been extended by a number of investigators,[4-6] to volume, time, and volume-time in a quite rigorous fashion."

1. **Zuber N & Findlay J A.** *The effects of non-uniform flow and concentration distributions and the effect of the local relative velocity on the average volumetric concentration in two-phase flow.* Schenectady, NY: General Electric Company, 1964. Report GEAP-4592.
2. **Wallis G B.** Some hydrodynamic aspects of two-phase flow and boiling. *International developments in heat transfer.* New York: American Society of Mechanical Engineers, 1961. Vol. 2. p. 319-40.
3. **Neal L G.** *An analysis of slip in gas-liquid flow, applicable to the bubble and slug flow regimes.* Kjeller, Norway: Kjeller Research Establishment, 1963. Report KR-62.
4. **Ishii M.** *Thermo-fluid dynamic theory of two-phase flow.* Paris: Evrolles, 1975. 248 p.
5. **Delhaye J M & Achard J L.** On the use of averaging operators in two-phase flow modeling. (Jones O C & Bankoff S G, eds.) *Thermal and hydraulic aspects of nuclear reactor safety. Vol. 1. Light water reactor safety.* New York: American Society of Mechanical Engineers, 1977. p. 289-332.
6. **Slattery J C.** *Momentum, energy, and mass transfer in continua.* New York: McGraw-Hill, 1972. 679 p.