# This Week's Citation Classic

Day M M. Amenable semigroups. *Ill. J. Math.* 1:509-44, 1957.
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This paper described the state of the topic of semigroups with invariant means, named the subject, gave a useful characterization theorem, and showed the relationships with many other interesting mathematical topics. [The *SCI®* indicates that this paper has been cited in over 110 publications since 1961, making it the most-cited paper ever published in this journal.]

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"In 1929, von Neumann studied a new class of groups, those with invariant means on the bounded functions.[1] My thesis (1939) studied semigroups with invariant means; thereafter, I worked in the field alternately with the geometry of Banach spaces. I finished a large geometrical project in 1955 and turned back to invariant means; in order to talk to my students I invented the term 'amenable (pronounced amean'able) semigroups.'

"For a while around 1960, it appeared that every property of semigroups which would appeal to an analyst (as contrasted with an algebraist) was equivalent to amenability. This is not true, but the number of conditions related to amenability is astonishing, ranging as it does from fixed point theory to spectral theory to statistics. (See my review paper[2] for semigroups and Bondar and Milnes[3] for groups.)

"My 1957 paper showed that amenability of a semigroup could be characterized by approximation to invariance in the semigroup algebra. In 1972, B.E. Johnson[4] showed that this, in turn, could be characterized in terms of derivations of the algebra. Thus he opened the new subject of amenable algebras, applications of which return to von Neumann's long-term interest, the operator algebras in Hilbert space now called von Neumann algebras.

"Following rashly the trail blazed by H. Pétard[5] in his study of the mathematical theory of big game hunting, I offered a framework for the mathematical theory of citation hunting. Consider (M)athematical, (R)hetorical, (PR)omotional, (C)hronological, and (PS)ociological qualities; these refer, respectively, to the content, clarity, publicity, timeliness, and charm of the paper. The validity of the scheme is evident: 'Amenable semigroups' scores well in all categories. (M) The paper is a report, definitive in 1957, of a good solid subject; any field begun by Hausdorf, Banach, and von Neumann has substance and ramifications beyond the ordinary. (R) The paper contains much more expository material than is common in papers appearing (then or now) in journals devoted to mathematical research. (PR) Favorable, very full reviews appeared in both *Mathematical Reviews*[6] and *Zentralblatt für Mathematik*.[7] (C) The paper appeared at an auspicious time when floods of new mathematicians had training and eagerness to attack these problems. (PS) The apt appellation 'amenable semigroups' snuggles subtly in the subconscious.''

1. von Neumann J. Zur allgemeine Theorie des Masses. *Fund. Math.* 13:73-116, 1929.
2. Day M M. Semigroups and amenability. (Folley K W, ed.) *Semigroups: proceedings of a symposium.* New York: Academic Press, 1969. p. 5-53.
3. Bondar J V & Milnes P. Amenability: a survey for statistical applications of Hunt-Stein and related conditions on groups. *Z. Wahrscheinlichkeitstheor.* 57:103-28, 1981.
4. Johnson B E. Cohomology in Banach algebras. *Mem. Amer. Math. Soc.* 127:1-96, 1972.
5. Pétard H. The mathematical theory of big game hunting. *Amer. Math. Mon.* 45:446-7, 1938.
6. Henriksen M. Review of "Amenable semigroups" by M.M. Day. *Math. Rev.* 19:1067, 1958.
7. Dinkler J. Review of "Amenable semigroups" by M.M. Day. *Zbl. Math.* 78:294, 1957.