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This Week's Citation Classic

Van Everdingen A F & Hurst W. The application of the Laplace transformation to flow problems in reservoirs.
Trans. Amer. Inst. Min. Met. Eng. **186**:305-24, 1949.
[Shell Oil Co., Houston, TX]

Two independent solutions of the material-balance equation are presented and unit functions for each solution are defined and computed using the Laplace transformation. Values are given over a wide range of conditions and examples of superposition are included. [The *SCI*® indicates that this paper has been cited in over 130 publications since 1961.]

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"Although trained as a mining engineer (Delft, 1923), my first experience with oil wells did not occur until 1928 when Shell transferred me to Sumatra. Much of the knowledge taken for granted today had yet to be discovered, and many of the opinions held at the time would now be considered nonsense. For instance, it was believed that a water drive resulted from surface water entering the formation at much higher elevations; that water standing in wells would damage an oil-bearing formation; and that it was necessary, when possible, to eliminate water production to conserve energy. The presence of connate water was not even recognized until the early-1930s. Pressure measurements were rare, but curiosity prompted me to try to find a relationship between pressure and production rate.

"During the early-1930s, great strides were made in understanding the behavior of oil reservoirs. The material-balance equation describing the flow of fluids in porous media (assuming low compressibility) was formulated. Its application to fluid flow around a well resulted in a linear partial differential equation of the second order.

"That much was known to me by 1944 when I learned that William Hurst had left his position with Humble and was looking for work in Houston. Hurst had already published his article, 'Water influx into a reservoir and its application to the equation of volumetric balance.'[1] However, this article was difficult to digest unless one was familiar with the Fourier-Bessel series. Recognizing Hurst to be a first-class mathematician, I arranged to have him hired at Shell and we began to study the Fourier-Bessel series in order to obtain simpler ways of computing and presenting the solutions to the material-balance equation. Henry Rainbow, also at Shell, suggested that we apply Laplace transformations to the equation. This suggestion enabled us to define and derive the two independent solutions, which we designated as the $P_{(t)}$ and $Q_{(t)}$ functions. As a simplification, we combined the five characteristics of oil, water, and reservoir into one constant and obtained what we referred to as dimensionless time. Because we had only electric desk-top calculators in those days, we spent most of the next four years computing $P_{(t)}$ and $Q_{(t)}$ values for sufficient dimensionless times to cover most cases encountered in actual practice. Therefore, our paper, which included these computations in tables and figures, was not published until 1949.

"It should be noted that it took a few years after publication for the value of those functions to 'sink in' and become recognized industrywide. Eventually, both Hurst and I were awarded Anthony F. Lucas Gold Medals. I believe the paper has been cited so often because it clearly defines the $P_{(t)}$ and $Q_{(t)}$ functions, which are the only solutions that satisfy inside and outside boundary conditions required by the differential equation. The only other function that has been used in pressure buildup work is the -Ei function, which is the point source solution; however, this solution can be used at small times only if one can ignore boundary conditions."

1. **Hurst W.** Water influx into a reservoir and its application to the equation of volumetric balance. *Trans. Amer. Inst. Min. Met. Eng.* **151**:57-72, 1943.