CC/NUMBER 9
FEBRUARY 28, 1983

This Week's Citation Classic

Happel J. Viscous flow in multiparticle systems: slow motion of fluids relative to beds of spherical particles. *AIChE J.* **4**:197-201, 1958.
[New York University, New York, NY]

A mathematical treatment is presented based on the notion that two concentric spheres can serve as the model for a fluid moving relative to a random assemblage of spherical particles. The outer sphere consists of the boundary of a fluid envelope with a 'free surface.' [The *SCI*® indicates that this paper has been cited in over 110 publications since 1961, making it one of the ten most-cited papers published in this journal.]

John Happel
Department of Chemical Engineering
and Applied Chemistry
Columbia University
New York, NY 10027

January 14, 1983

"This paper was written just 100 years after Henry Darcy[1] published his studies showing that the rate of flow of water through sand bed filters is proportional to the pressure drop. I was chairman of the chemical engineering department at New York University and was intrigued by the fact that, although a great many studies had been devoted to experimental determination of the Darcy permeability of various types of porous media to fluid flow, there was no purely theoretical prediction that required no experimental data even for idealized systems like beds of spherical particles. My brilliant doctoral student, Howard Brenner, addressed this question and found that a rigorous treatment was remarkably complicated. I decided to explore a variation of the well-known cell models in which a bed of particles is assumed to consist of identical compartments with one particle occupying each, in the hope that an idealized simple technique might provide useful insight. Surprisingly, the result showed close agreement with experimental data over a wide range of bed porosity for regular shaped particles. When I reported this to Howard, thinking that this would have saved him time, he facetiously remarked that it was too bad I hadn't discovered this while he was engaged in his thesis research.

"One reason this paper has been frequently cited is that the term 'free surface model' is readily visualized. The derivation is based on a simple solution of the Stokes-Navier equations of motion, postulates a clearly defined and readily acceptable boundary value problem, and gives an exact prediction of the Darcy permeability with no additional empirical assumptions.

"Another reason is the fact that the behavior of systems involving the motion of fluids relative to aggregates of particles covers a wide range of phenomena of interest to both scientists and engineers. For chemical engineers, fluid dynamic models often furnish the basis for more extended investigations involving other transport phenomena, coupled in some cases with chemical reactions. It is advantageous to start with a relatively simple model before introducing the complications posed by these additional variables. For example, Kumar and Sirkar[2] have more recently studied the application of cell models to mass transfer in a cloud of size distributed droplets.

"A few years after this publication appeared, we wrote a book[3,4] that collected research in theoretical studies involving particulate media at low Reynolds number fluid flow. We ventured to predict that studies of this type would become increasingly important for many phenomena of interest to technologists. This area of research has in fact burgeoned in recent years and I would like to believe that some of this interest stems from the work of my colleagues and students in the 1950s."

1. **Darcy H P G.** *Les fontaines publiques de la ville de Dijon.* Paris: Victor Dalmont, 1856. 647 p.
2. **Kumar S & Sirkar K K.** Creeping flow mass transfer in a cloud of size distributed round drops at high Schmidt numbers. *Chem. Eng. Sci.* **34**:1231-42, 1979.
3. **Happel J & Brenner H.** *Low Reynolds number hydrodynamics with special applications to particulate media.* Englewood Cliffs, NJ: Prentice-Hall, 1965. 553 p.
[The *SCI* indicates that this book has been cited in over 645 publications since 1965.]
4. ----------------------------, *Low Reynolds number hydrodynamics with special applications to particulate media.* Leyden: Noordhoff International Publishing, 1973. 569 p.
[The *SCI* indicates that this book has been cited in over 140 publications since 1973.]