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This Week's Citation Classic

Box G E **P** & **Wilson K B.** On the experimental attainment of optimum conditions. *J. Roy. Statist. Soc. Ser. B Metho.* **13**:1-45**,** 1951.
[Imperial Chemical Industries, Dyestuffs Division Headquarters, Blackley, Manchester, England]

The problem is discussed of finding experimentally the levels of a number of quantitative variables at which some dependent response has a maximum value. Experimental designs and procedures are introduced for determining a path of steepest ascent and for exploring maxima and ridges. [The *Science Citation Index*® (*SCI*®) and the *Social Sciences Citation Index*® (*SSCI*®) indicate that this paper has been cited over 325 times since 1961.]

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"In 1948 I went to work as a statistician for Imperial Chemical Industries at a research center near Manchester, England. I had been told that a problem of great importance to the company was that of finding optimal operating conditions (for example, the temperature [T], pressure [P], and the concentrations of the various reactants which gave maximum yield [y] of the desired product). I was introduced to a chemist, K.B. Wilson, who had recently suggested that the problem might be tackled using the method of steepest ascent. This required that derivatives $\partial y/\partial T$, $\partial y/\partial P$, etc., should be determined as accurately as possible in the face of inevitable experimental error. This could be done using fractional factorial and other orthogonal designs.

"After some encouraging successes with laboratory scale experiments on previously unexplored systems, the opportunity arose to try our methods on an important full-scale process that had been operating for some time. At first our results were disappointing. But it was eventually realized that the 'one factor at a time' experimentation that was then routinely used in the development of processes would probably have already brought the first derivatives $\partial y/\partial T$, etc., close to zero, so that, in this context, ascent methods used alone were unlikely to produce much improvement. However, it was noticed that very large interactions usually occurred between variables such as reaction time, reaction temperature, and reaction pressure, suggesting that the surfaces we were studying contained oblique ridges. And it was later realized that this ought to be expected from kinetic considerations. The approximate nature of these surfaces could be determined by augmenting our designs in a way that allowed all the second derivatives $\frac{\partial^2 y}{(\partial T)^2}$, $\frac{\partial^2 y}{\partial T \partial P}$, etc., to be additionally estimated. Methods of analysis which could show the nature of the resulting ridges and so lead to their exploitation to provide better processes were then worked out. The technique we had thus been brought to—first order steepest ascent followed by second order local exploration—was now tried on a number of process improvement studies and proved to be extremely effective in practice.

"At the suggestion of George A. Barnard, then of Imperial College, London, Wilson and I submitted a paper about our work to the research section of the Royal Statistical Society. My understanding is that the referees for this paper failed to agree, one thinking it very good, the other very bad. But it was accepted and read to the Society in 1950, where it provoked a spirited and encouraging discussion.

"I believe the publication is frequently cited because it was the beginning of what has come to be called response surface methodology, which has been the subject of much further research. Furthermore, these methods have since been applied in a very wide variety of fields of application. A recent, general reference is *Statistics for Experimenters.*"[1]

1. **Box G E P, Hunter WG & Hunter J S.** *Statistics for experimenters.* New York: Wiley, 1978. 653 p.