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This Week's Citation Classic

Gilbert E G. The decoupling of multivariable systems by state feedback. SIAM J. Contr. 7:50-63, 1969.
[Computer, Information and Control Engineering, Univ. Michigan, Ann Arbor, MI]

Let A, B, C,F,G be respectively n x n, n x m, mxn, mxn, mxm real matrices. The open-loop system $\dot{x} = Ax + Bu$, $y = Cx$ is decoupled by the feedback law $u = Fx + Gv$ if the closed-loop transfer function $H(s) = C(Is - A - BF)^{-1}BG$ is diagonal. The class of decoupled closed-loop systems is described and related parametrically to the class of decoupling feedback laws. In general, some characteristic roots of A + BF must be fixed, but the rest can be assigned arbitrarily. [The SCI® indicates that this paper has been cited over 115 times since 1969.]

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"I first became interested in decoupled feedback systems around 1959 when the first papers involving transfer-function methods came out. These papers had a serious defect: the required algebraic manipulations led to the 'cancellation' of system dynamics. Because the system cancellations were not well understood, system instability was overlooked. This motivated me to explain the cancellation effects in terms of controllability and observability.[1] But attempts to exploit the explanation in design proved clumsy and ineffective. B.S. Morgan was working with me on his doctoral research at that time and we wondered if decoupling by state feedback might be possible, since such an approach would avoid the troublesome manipulation of transfer functions. Morgan was able to exhibit a class of decoupling feedback laws when the open-loop system satisfied a special condition [2] Later, Z.V. Rekasius extended Morgan's results somewhat,[3] but a complete solution remained beyond reach.

"After Morgan's work my interest in the problem waned. In 1966, PL. Falb was at the University of Michigan and he explained to me important new results that he and W.A. Wolovich had obtained.[4] They included a necessary and sufficient condition for the existence of a decoupling feedback law. But they did not describe the whole class of decoupling laws and the relationship between a decoupling law and the characteritics of the resulting closed-loop system. The unanswered questions renewed my interest. From Falb and Wolovich[4] it was obvious to me that they could be answered if I considered a special class of open-loop systems which I called integration-decoupled systems. By a special choice of coordinates these systems were represented by what I called canonically decoupled systems. From the properties of canonically decoupled systems the key results were evident. What led me to the canonically decoupled systems? It was not a flash of insight based on my long involvement with the problem. It was just hard work; I tried many examples until the necessary structures appeared. Why were the results not obtained five years earlier? Because I didn't think enough about feedback invariants and I didn't ask obvious questions which had to have answers. After my paper, many other approaches to the decoupling problem appeared. The most elegant of these is the geometric theory of W.M. Wonham and AS. Morse,[5] which also has other applications.

"I have some conjectures on why the paper is frequently cited: it concerns the important problem of designing feedback controls for large-scale systems; it presents the first complete solution to the decoupling problem; the presentation provides insight and is self-contained and accessible; and a convenient and well-documented computer program is available[6] for applying the theory to specific designs."

1. Gilbert E G. Controllability and observability in multivariable control systems. SIAM J. Contr. 1:128-51, 1963.
2. Morgan B S. The synthesis of linear multivariable systems by state variable feedback. 1964 Joint Automatic Control Conference at Stanford University, June 24. 25. 26: preprints of conference papers. New York: American Automatic Control Council, 1964. p. 468-72.
3. Rekasius Z V. Decoupling of multivariable systems by means of state feedback. (Van Valkenburg M E, ed.) Proceedings: Third Annual Allerton Conference on Circuit and System Theory, 20-22 October 1965. Monticello. Illinois. Urbana, IL: University of Illinois, 1965. p. 439-48.
4. Falb P L & Wolovich W A. Decoupling in the design and synthesis of multivariable control systems. IEEE Trans. Automat. Contr. AC-12:651-9, 1967.
5. Wonham W M & Morse A S. Decoupling and pole assignment in linear multivariable systems: a geometric approach. SIAM J. Contr. 8:1-18, 1970.
6. Gilbert E G & Pivnichny J R. A computer program for the synthesis of decoupled multivariable feedback systems. IEEE Trans. Automat. Contr. AC-14:652-9, 1969.