

Zubarev D N. *Double-time Green Functions in statistical physics.*  
*Usp. Fiz. Nauk. SSSR* 71:71-116, 1960; *Sov. Phys. Usp.* 3:320-45, 1960

In the paper a brief account is given of the temperature-dependent double-time Green functions and the simplest applications to the theory of irreversible processes, to the theory of superconductivity, to ferromagnetism, and so on. In the applications, the double-time retarded and advanced Green functions which were introduced by Bogolyubov and Tyablikov<sup>1</sup> in the discussion of thermodynamic properties of systems are used. [The SCI® indicates that this paper has been cited over 740 times as *Usp. Fiz. Nauk. SSSR* and over 875 times as *Sov. Phys. Usp.* since 1961.]

D.N. Zubarev  
V.A. Steklov Institute of Mathematics  
Academy of Sciences of the USSR  
Vavilov Street 42  
Moscow, 117966, GSP-1  
USSR

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"It is gratifying to learn that my 1960 article has been so frequently cited, but it is even more encouraging to consider the progress in the application of Green functions in statistical physics and especially in solidstate physics. Green functions in statistical mechanics are a convenient generalization of the concept of correlation functions. Like the latter they are intimately connected with the calculation of observable quantities, but have advantages in construction and solution of the equations determining them. The doubletime Green functions are defined as the average values of some product of dynamical variables (or operators) which represent the change of the average value of one of them due to the switching of the perturbation.

"The Green functions applied in statistics differ from the field theory Green functions only in the method of averaging. Instead of averaging over the lowest vacuum state of the system, the averaging is performed over

the Gibbs grand canonical ensemble. Consequently, the Green functions depend both on time and on temperature.

"For many problems in statistical mechanics, we can confine ourselves to two time functions, since for them we can use simple spectral expansion. On the other hand they contain a sufficiently large amount of information on equilibrium and nonequilibrium properties of many body systems.

"The retarded Green function is equal to the change of the average value of some dynamic quantity by the time  $t$  due to the instantaneous switching on of the perturbation at the time  $t_1$ . The relations for linear response of a system are called the Kubo formulas. The dynamic susceptibility of charged particles in alternating electromagnetic field, dielectric permittivity, electric conductivities, and dielectric polarizability is simply connected to retarded Green functions. The kinetic coefficients, thermal conductivity and viscosity, can also be represented as correlation functions of flux densities (Green-Kubo-formulas) or as retarded Green functions.<sup>2</sup> The poles of single-particle Green functions on the real energy axis represent the elementary excitation spectrum. In the real systems there is a cut along the real energy axis, and the different Fourier component of some operator (mass-operator) in upper and lower half-plane of complex values of energy gave the damping of elementary excitation.

"It is amusing to remember that this article was at first rejected by the editorial board as 'too mathematical.' The inclusion of it on the list of most cited papers, I think, is due mainly to the simplicity of its form and the favourable moment of publication: there was an imperative need for such simple methods in the very complicated problems of solid-state physics. As further development shows, the method of double-time Green functions appears to be very powerful in solving many problems in the theory of condensed matter.<sup>3-5</sup> A great number of scientists have employed this method to their special problems. In my opinion this is the reason why my 1960 article—the first review of this method—has been so highly cited."

1. Bogolyubov N N & Tyablikov S V. Retarded and advanced Green functions in statistical physics. *Dokl. Akad. Nauk. SSSR* 126:53-6, 1959; *Soviet Phys. Doklady* 4:589-93, 1959.
2. Zubarev D N. *Nonequilibrium statistical thermodynamics*. New York: Plenum, 1974. 489 p.
3. Bonch-Bruевич V I & Tyablikov S V. *The Green function method in statistical mechanics*. Amsterdam: North-Holland, 1962. 251 p.
4. Tyablikov S V. *Methods in the quantum theory of magnetism*. New York: Plenum, 1967. 354 p.
5. Stinchcombe R B. Formulation of response theory. (Halley J W, ed.) *Correlation functions and quasiparticle interaction in condensed matter*. New York: Plenum, 1977. p. 3-44.