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This Week's Citation Classic

Bradshaw P, Ferriss D H & Atwell N P. Calculation of boundary-layer development using the turbulent energy equation. *J. Fluid Mech.* **28:**593-616, 1967. [Aerodynamics Div., Natl. Physical Lab., Teddington, England]

The turbulent energy equation was, and is, better documented experimentally than the shear-stress transport equation. We used empirical relations between the shear stress and the terms in the former equation to produce a soluble transport equation for shear stress. [The *SCI*® indicates that this paper has been cited over 190 times since 1967.]

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"Alan Townsend[1] used a (partly-empirical) version of the (exact) turbulent energy equation to describe the inner layer of a boundary layer, neglecting the 'advection' term to convert a partial differential equation into an ordinary one. To predict the whole boundary layer, I had to (1) retain the advection term, (2) think up empirical relations (based on experimental data) for the other terms, and (3) solve the resulting PDEs. The relations in step (2) were between turbulence properties only, avoiding any 'eddy viscosity' assumptions relating turbulence to mean flow (which would actually contradict the exact equations).

"Dave Ferriss and I programmed a simple explicit scheme for KDF9, a paper-tape-oriented computer which the NPL retired only recently. Dave kept a paper-tape winder on his desk, and I used to fiddle with it while we discussed our problems: finally Dave brought a set of Greek worry beads back from vacation and told me to fiddle with those instead. The explicit scheme was mildly unstable, but Peter Williams pointed out that the equations were hyperbolic, rather than diabolic as we were beginning to think, so that the method of characteristics could be used. Norman Atwell, a college student who came to us for six months 'industrial experience' as part of his degree course, did the mathematics. The changeover was quite easy, and we had the method in publishable form after a total of about nine months' work.

"Ours was the first method that completely avoided eddy viscosity assumptions and, instead, made extensive use of turbulence measurements. Other 'transport equation' methods, some at least of which were developed quite independently, appeared a little later. Still other workers were developing methods which used eddy viscosity assumptions in a somewhat more subtle way, following a suggestion made by Prandtl.[2] Since then, most new methods have been based on one or other of these alternatives: our transport-equation approach ought to be more accurate but relies more on (inaccurate!) experimental data.

"Since these two schools of thought still exist,[3] at least half of the citations of our paper must have been unfavourable. However, it is satisfying to have helped stimulate the application of modern computers to turbulence calculations, even if some of the more recent calculation methods spread the experimental data thinly, over a large area of equations. Complete time-dependent solutions for the fluctuating motion itself need too much computer time and storage to be useful for engineering purposes at present: however they need no empirical data, in principle, and will probably start to put both the experimenters and the developers of empirical calculation methods out of business in 20 years or so. I reach retiring age in the year 2000 anyway."

1. **Townsend A A.** Equilibrium layers and wall turbulence. *J. Fluid Mech.* **11:**97-116, 1961.
2. **Tollmien W,** ed. *Ludwig Prandtl Gesammelle Abhandlunger.* Heidelberg: Springer-Verlag, 1961. p. 874.
3. **Bradshaw P, Cebeci T C & Whitelaw J H.** *Engineering methods for the calculation of turbulent flow.* London: Academic Press. In press, 1980.