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This Week's Citation Classic

Bryson A E, Jr. & Ho Y C. *Applied optimal control: optimization, estimation, and control* Waltham, MA: Blaisdell, 1969. 481 p. [Stanford Univ., Stanford, CA and Harvard Univ., Cambridge, MA]

The research that led to the writing of this book was stimulated by real design problems encountered in working full-time at the Hughes R&D Labs (1951-53) and later as a consultant to the Raytheon Co. while on the faculty at Harvard University. The help and enthusiasm of many colleagues and students gradually showed us how to solve some of these interesting optimization problems. [The *SCI*® indicates that this book has been cited over 520 times since 1969.]

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"A Hughes air-to-air missile was being flown in 1952 at a desert testing range. The range-safety officer asked how far the missile could glide after rocket motor burn-out if the guidance system failed. If we applied the classical glider solution (fly at maximum L/D) to the missile, it would climb steeply and do a loop! Richard H. Edwards, my colleague at Hughes (now a professor at the University of Southern California), showed that this maximum range problem was a problem in the calculus of variations and he formulated it in terms of Lagrange multiplier functions. Attempts to solve this two-point boundary-value problem on the best analog computers of that day ended in failure.

"In 1954, Raytheon was doing preliminary design on a ground-to-air missile. The problem arose of how to program the thrust as a function of time to achieve maximum horizontal range at a specified final altitude. This problem was solved by Stanley E. Ross in his PhD dissertation at Harvard in 1957 using a digital computer.[1] This was followed at Raytheon by Walter F. Denham's determination of the flight path of a lifting entry vehicle that minimizes aerodynamic heating using the new IBM 704 digital computer.[2]

"The problem of determining the flight path of an airplane for minimum time-to-climb to a specified altitude arose in connection with Raytheon's air-to-air missile which is carried on a supersonic interceptor. This problem was solved at Raytheon in 1961 by Denham.[3]

"The digital computer was the key element that made it possible to find solutions to realistic two-point boundary-value problems. However, another important element was the development of efficient algorithms. Gradient algorithms were developed independently by Henry J. Kelley and by our group at about the same time (1960-1961).[3,4]

"Having had some success in solving realistic problems, Yu-Chi Ho and I offered a two-week course on dynamic optimization at Harvard in the summer of 1963. This was attended by 100 engineers, most of whom were from industry. In 1967, we offered an intensive two-day continuing-education course sponsored by AIAA which featured simultaneous use of two viewgraphs. This course was repeated several times and is still available from AIAA in the form of audio-cassettes and a slide-book. We began offering a graduate course at Harvard in 1963, which was also given at MIT in 1966. As a result of the fine reception given to these courses, we were encouraged to extend our class notes and put them into book form.

"Speculating on why the book has been frequently cited, we believe it may be because it is oriented toward solving realistic problems and contains examples and exercises that are closely related to real engineering problems."

1. **Bryson A E, Jr. & Ross S E.** Optimum rocket trajectories with aerodynamic drag. *Jet Propulsion* **28**:465-9, 1958.
2. **Bryson A E, Denham W F, Carroll F J & Mikami K.** Determination of lift or drag programs to minimize re-entry heating. *J. Aerospace Sci.* **29**:420-30, 1962.
3. **Bryson A E, Jr. & Denham W F.** A steepest-ascent method for solving optimum programming problems. *Trans. ASME* **29E**:247-57, 1962.
4. **Kelley H J.** Methods of gradients. (Leitmann G. ed.) *Optimization techniques.* New York: Academic Press, 1962. p. 206-54.