

# This Week's Citation Classic

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Bates D R & Damgaard A. The calculation of the absolute strengths of spectral lines. *Phil Trans. Roy. Soc. London A* 242:101-22, 1949.

It is shown that in calculating the radial transition integrals which determine the absolute strengths of spectral lines it is usually a sufficient approximation to neglect the departure of the potential of an atom or ion from its asymptotic Coulomb form. This enables an analytic expression for the integral to be derived. Tables are presented from which the absolute strengths of spectral lines can be obtained by interpolation if the energies of the initial and final levels are known. The method gives remarkably accurate results. [The *SC<sup>i</sup>* indicates that this paper has been cited 555 times since 1961.]

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"In 1946, when I was a Mathematics Lecturer at University College, London under Professor H. S. W. Massey, I became aware that astrophysicists needed to know the absolute strengths of many spectral lines. There was then little reliable information on these strengths either from experiment or from theory. I therefore explored the possibility of computing strengths by the Hartree or Hartree-Fock approximations.

"Fortunately, the only aid I had to complete the heavy numerical work entailed was a desk calculator (the differential analyser which Professor Massey had brought from Belfast to London having been destroyed during an air raid). It soon became apparent that the use of neither of the approximations would be practicable. For every spectral line the equations appropriate to the initial and final wave functions had to be solved by an outward step-by-step integration from the origin: and there then followed a numerical quadrature. The step-by-step integrations occupied most time. Moreover, each had to be done repeatedly because the wave equation contains a characteristic energy which if not chosen correctly leads to the solution obtained diverging at large radial distances, and the correct choice can only be found by an integrative procedure. Frustratingly, the choice which avoided the solution diverging in the most important region was not quite equal to the term energy. Instead

of proceeding outwards from the origin, one should clearly proceed inwards toward the origin taking the characteristic energies to be the observed term energies. This ensures accurate solutions at large radial distances and the inevitable divergence at the origin could doubtless be circumvented in some way.

"At this stage I was joined by Agnete Damgaard, my first research student. On examining a few representative cases we discovered that the line strength was scarcely affected by the contribution from the region where the field on the active electron differed appreciably from its asymptotic Coulomb form. This enabled us to adopt standard asymptotic expansions for the required solutions to the wave equations and to do the final integrations analytically avoiding the divergence at the origin by a judicious truncation of the series obtained. We found that the results could be expressed as a product of two functions, one simple, the other such that for a particular class of transition it may, on combining the parameters suitably, be presented in a doubly linear interpolation manner which allows linear interpolation. "Before proceeding with the systematic computations it would have been prudent for us to treat all transitions on which measurements had been made. We did not do so because we had confidence in the method and because I was unwilling to treat special cases which would delay the completion of the work (being selfishly overconscious of the waste of my scientifically creative years by the war). After the tables were prepared we had the satisfaction of using them to show our confidence to have been justified. "It is natural that our method has been widely applied in that it enables line strengths, which are often needed, to be easily determined from known term values. For many years computer programs have been more convenient than our original tables. Moreover, compilations of individual line strengths, based mainly on our method, have been published. That our paper continues to be cited so often is illustrative of the punctilious code of scientists. "I still get an occasional request for a reprint.

"How strange that anyone could be optimistic enough to hope that such a request could be met after more than a quarter of a century!"