NUMBER 18
APRIL 30, 1979

This Week's Citation Classic

Tinney W F & Walker J W. Direct solutions of sparse network equations by ptimally ordered triangular factorization. *Proc, IEEE* **55**:1801-9, 1967. [Bonneville Power Administration, Portland, OR]

A new method is given for direct solution of large sparse systems of linear equations. The essentials are: (1) store and process only non-zero terms, (2) factor the coefficient matrix into upper and lower triangular matrices, (3) order the factorization to approximately minimize new non-zero terms, (4) use the factors to obtain the direct solutions by forward and back substitution. [The *SCI*® indicates that this paper has been cited over 100 times since 1967.]

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September 13, 1978

"Today, awareness of sparse matrix methods is practically inescapable. Papers, books, and symposia on sparsity abound and applications exist in almost every field. But only a decade ago, when our paper was published, the concepts of sparsity were virtually unknown. The advantages are so great (10 to 100 or more) and the idea is so simple, it is puzzling that it took so long to be discovered.

"The aim of sparsity methods is to exploit the property that the coefficients of many large systems of simultaneous equations are mostly zero. Standard matrix methods take no advantage of the zeros, storing and processing them the same as non-zero numbers, and causing most, or all, of them to become non-zero in the solution process.

"Working on computer solution of large network problems afforded us an ideal opportunity to stumble onto the sparsity concept. We had developed effective sparsity methods by 1961 and published a paper on the subject in 1963.[1] It was completely ignored. A paper on sparsity appeared as early as 1957 in Management Science.[2] It had also been ignored and was unknown to us Several others had independently begun working on sparsity by the mid 60s Our work began to be noticed in about 1967 when we were invited to submit the paper that prompted this commentary. The critical turning point in recognition of sparsity was the first Sparse Matrix Symposium, held in 1968 by IBM, in which our work and that of a few others was presented After this event, which drew people from many fields, work on sparsity took off with astonishing speed.

"If sparsity is simple, what was left to be done? Follow-on activity has included customizing techniques for each application field, finding new applications and developing efficient algorithms. These are complex problems that continue to challenge the best analysts. Gratifyingly, the ideas in our paper remain valid in spite of the many advancements.

"The history of sparsity is an interesting example of a development whose time was long overdue. Why did it take so long to discover and catch on? The answer might shed light on similar occurrences of technological blindness. Since the possibilities of sparsity exploitation should have been self-evident to a village idiot, it is perplexing how they escaped notice by aplied mathematicians. As it turned out, sparsity was pioneered by a scattering of problem solvers, not mathematicians. This suggests that similar seemingly obvious ideas may still be escaping detection, not because of their obscurity but because of their elusive simplicity.

"Our paper of 1967 appeared at just the right time and place to provide a convenient reference for the explosion of work on sparsity that followed. If we had had an inkling of its potential impact, we would have written it quite differently "

1. **Sato N & Tinney W F.** Techniques for exploiting the sparsity of the network admittance matrix. *IEEE Trans. Power App. Syst.* **82**:944-50, 1963.
2. **Markowitz H M.** The elimination form of the inverse and its application to linear programming. *Manage. Sci*. **3**:255-69, 1957.