Molecules consist of atoms, and for our purposes the actual crystal structure could be replaced by an idealized one consisting of discrete, non-vibrating, point atoms. In this way the phase problem was transformed from one having no solution in principle to one that was greatly overdetermined because the number of observed intensities usually exceeds by far the number of parameters (atomic position coordinates) needed to fix the structure.

Recognition of the overdetermination of the problem suggested the use of probabilistic techniques for its solution. The unknown atomic position vectors were assumed to be random variables, uniformly and independently distributed. The unknown phases, as functions of the primitive random variables, were themselves random variables. The conditional probability distribution of selected linear combinations of the phases (the so-called structure invariants) could then be found. In favorable cases the distributions lead to reliable estimates for the structure invariants and the latter, as certain well-defined linear combinations of the phases, lead to unique values for the phases.

The result of the central role of the structure invariants that serve to link the desired phases with the measured intensities is the solution of the phase problem. Recognition of the overdetermination of the number of parameters (atomic positions) needed to fix the structure.

Thus, I joined my colleague, Jerome Karle, at the Naval Research Laboratory in 1947, the phase problem was still the central crystallographic X-ray crystallography— it had remained unsolved and was regarded as unsolvable. We soon discovered that the key to its solution was the known property of real crystals, atomicity.