Available kinds of straight lines are listed with their mathematical characteristics and examples of their application. Choice of a line can be influenced by its proposed use—for prediction or for trend description—and also by the type of data at hand. (The SCI® indicates that this paper has been cited in over 295 publications since 1973.)

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I am not surprised that this paper has been frequently cited because I received about 1,500 requests for reprints from more than 50 countries—far more than for anything else I had published. The reason is simply that the paper treats one of the commonest procedures in science: fitting a straight line to data. One would have thought that so fundamental a matter would be adequately treated in elementary texts. Actually, these books usually give details of least-squares fitting for only the two classical situations: 1) the \( Y \) on \( X \) regression when \( X \) is without measurement error or natural variability and 2) the \( Y \) on \( X \) and \( X \) on \( Y \) regressions as predictive lines for bivariate normal (binormal) arrays where measurement errors are negligible but there is mutual natural variability (MV)—defined as natural variability that cannot be identified with either \( Y \) or \( X \) separately and cannot be divided quantitatively between them. What textbooks usually lack is a clear distinction between predictive and descriptive lines and discussions of procedures for dealing with three situations that occur constantly. These situations occur when 1) the sample available has been drawn from a real but nonbinormal population, or 2) the sample has not been drawn randomly from the population, or 3) the population is artificial and of indefinite extent.

There is still disagreement among statisticians about a descriptive central trend line for MV binormal data. Some claim, in effect, that it is impossible to locate such a line. However, as early as 1916, the Norwegian oceanographer H. Sverdrup\(^1\) proposed using the geometric mean regression (GMR) of the two ordinary regressions, which is also the ratio of the standard deviations of \( Y \) and \( X \) and is called variously the functional regression and the standard (or reduced) major axis of the array of observations. About 1940, the French crustaceologist C. Teissier\(^2\) brought the GMR to the attention of biologists. The GMR is in fact the only trend line available for binormal arrays that has both of the two essential characteristics: symmetry and scale invariance. Its critics complain that it lacks theoretical justification and must be regarded as an ad hoc procedure,\(^3\) but names don't matter if something fills a real need.

Recently, I elaborated further on the characteristics and applications of the GMR,\(^4\) comparing it with the ordinary major axis and other lines. The GMR is especially useful in the three nonclassical situations listed above. For them, it is usually the best available line not only for trend description but also for prediction, whereas ordinary regressions fail badly with nonbinormal MV arrays and incomplete samples. An interesting recent development is that J. Schnute\(^5\) has proposed a symmetrical and scale-invariant trend line based on a new principle, but unfortunately it is rather difficult to compute, and it does not apply to binormal distributions.