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Chapter 2

Probabilistic Semantics

Introduction

This chapter might as well be entitled "probabilistic semasiology." It occupies the central place in my system of judgments. Our principal task is to build a model reflecting both the logical structure of language and its complexity expressed *by the absence of one-to-one correspondence* between a symbol and its referent.

Communication between people takes place on a logical level. Chains of syllogisms are often interspersed with our everyday speech. Nobody formulates postulates explicitly, but we easily catch them in the simplest statements. The logical structure of speech is learned in childhood. For example, a woman insistently asks a little boy, the son of a mathematician, "Why aren't you a girl?" After a moment of pondering, he answers, "Probably because I am a boy." In this syllogism an axiom is used stating *that it is impossible to be a boy and a girl simultaneously*. Hence, the boy comes to the conclusion that since he is a boy he, therefore, cannot be a girl. But this judgment does not seem too profound even to a child, and for this reason he adds the word "probably."

Even tipsy people try to reason logically.

Once in a dining car I asked a young man why he was so gloomy. The answer was brief, "Because my wife has left me." And to the question, "Why has she left?" he gave another brief answer, "Why? Because I have not left her first." A hidden postulate in this syllogism sounds as follows: *the situation in the family demanded a divorce; hence the wife took this step only because the husband had not done so earlier.*

Certainly, in everyday speech not only logical means are used but also plausible reasoning, such as judgments based on analogy, felicitous illustrations, etc. But they are still of lesser weight: remember the French proverb, "Comparaison n'est pas raison." The fact that in everyday speech initial postulates are not explicitly formulated allows one to ascribe different weights to them, which, of course, enriches the system of judgments and broadens its limits, though this makes it less strict.

Black (1949) paid attention to this circumstance when he criticized strictly axiomatic means of constructing theories with their unconditional democratism in evaluating both axioms and the consequences from them. If not all consequences are equally important, some of them which are inconsistent with other observations or in opposition to them may be omitted, and this will not destroy the whole system of judgments. True, statements with only implicitly formulated premises are difficult to analyze critically. It is interesting to note that the most remarkable critical structures of the eighteenth and nineteeth centuries, like the philosophy of Kant or of Hegel, are built so that initial premises are not made explicit.

The logical structure of speech has attracted the attention of logicians. Studying language, they could not but pay attention to its logical insufficiency. Hence, a desire arose to rigorize language, especially that of science and philosophy. Thus the program of the neopositivists appeared, which I have already mentioned. And when this trend proved inconsistent and began to degenerate, it was replaced by *logical semantics*, whose origin, it is true, goes back to the works of Peirce (1839–1914) and Frege (1898–1925).

Logical semantics is a branch of metalogic dealing with the interpretation of strictly formalized statements (logical calculus) studying such problems as names, sense, meaning, truth, and falsity. With the help of these notions, logical semantics studies the means of expression of artificial and scientific languages. My task here is more modest: the diagnosis of language and not its therapy. One can readily observe profound genetic links of two trends, logical positivism and logical semantics. The same authors may in some of their papers adhere to both trends simultaneously. Methods and problems of logical semantics have turned out to be deeply related to those of mathematical logic. Great contributions to the development of logical semantics were made by Russell, Carnap, Church, Tarski, and Kemeny. Somewhere at the crossing of mathematical logic and theory of automata another trend has emerged, *mathematical linguistics*, which constructs strictly formalized models of natural and artificial languages. It is concerned with building strictly formalized grammars for symbolic languages.

Another alternative to a formal logical approach to language proceeded from the belief in the power and richness of the natural language, of its inexpressibility in the system of formal-logical constructions. In opposition to the statements of the neopositivists, here it was stated that the natural languages should not be drastically altered (every correction would impoverish them) but rather should be studied and used correctly. This trend had already originated at the beginning of the twentieth century in the philosophical milieu of Great Britain.

It was then that the question of correct comprehension of texts and the rational interpretation of paradoxical statements, abundant in Western philosophy, became especially acute.

The first steps toward the foundation of the British school of linguistic philosophy were made, as I have mentioned, by the British philosopher Moore at Cambridge. His widely known book *Principia Ethica* (Moore, 1903) was published in 1903, and was followed by another book, *The Conception of Reality* (Moore, 1959), in 1917. In the first of these books, he says that in ethics, as in other philosophical studies, difficulties and troubles arise for a very simple reason: from a desire to answer a question without taking the trouble to understand its sense. Philosophers often face more than one question: they confront complex issues.

One such polysemantic question is the central problem of ethics, "What is good?" Moore wrote that if he is asked how good is to be defined, the answer is that it cannot be defined and that is all he has to say about it, disappointing though this answer may appear. In the second book, Moore subjected to a linguistic analysis a paradoxical statement of the British philosopher Bradley, "Time is unreal," and showed that these words mean something quite different from what ordinary people would think they mean. Later, the statement that all troubles in philosophy are due to using words of everyday language in an arbitrary and uncommon sense became the burden of reasoning of British analytical philosophers.[1]

Another example will illustrate more clearly the way Moore subjected philosophical statements to a critical analysis. In analyzing the statement that the universe is spiritual, he remarked that too many different meanings are ascribed to the word "spiritual." If the universe is declared to be spiritual, he said, then ". . . chairs and tables, and mountains which seem to be very different from us, will be more like us than we think."

However, Moore, as opposed to the neopositivists, did not deny the value of metaphysics and did not demand a reform of language; he remained an analyst purporting to have a more profound comprehension of ideas and objects of investigation. He was reproached with making a fetish of the natural language and declaring it holy. [A more detailed de-

[1] Much earlier, the wrong usage of words in philosophy was dwelled upon by Helvetius (1758) in his tractatus *On Mind*. One of its chapters is entitled "On Wrong Use of Words." His arguments strikingly resemble the arguments of Moore.

scription of Moore's ideas can be found in a brief but highly readable article (Paul, 1956); what is written above is borrowed from this article.]

I shall not dwell here upon the progress the British linguistic school has made; it had many supporters in the United States as well. I shall confine myself to several remarks on Ziff's book *Semantic Analysis* (1964) published in the United States rather recently. It deals with the semantic analysis of statements not found in philosophical texts. In the preface, the author notes that his writing the book was inspired by the idea that it would be helpful to say what the phrase "good painting" meant. The author faced this question while working on a manuscript on aesthetics. There the following questions arose: "Why should anyone believe what I said?" and "What made me think it was so?" In the last chapter of the book (Ziff, 1964), 160 phrases containing the word "good" are subjected to a comparative semantic analysis. Here are three phrases from the list:

(7) This is a good strawberry.
(8) This is a good lemon.
(9) This is a good carving knife.

It is readily seen that the word "good" has quite different meanings in these phrases. A good lemon should be sour; a good strawberry, on the contrary, should not be sour; and a good carving knife should be sharp, which has nothing to do either with the quality of a lemon or with that of a strawberry. Ziff finishes his book with a statement that the meaning of the word "good" varies but is always associated with answering questions reflecting some special interest or value of ours.

In the papers of representatives of the British linguistic philosophical school, one always, or almost always, comes across sharp opposition of their views to those of the neopositivists. However, the paper by Strawson (1956) contains the statement that it is not clear whether philosophers trying to construct an artificial language and those engaged in analyzing the natural language should be regarded as two hostile camps; he suggests that they supplement each other to a certain degree, and often these two trends are unified under the title of analytical philosophy since they have a common goal—the analysis of language. It seems that we should go farther and formulate the problem more broadly. There is a need in a language model to reflect both its many-sided and illogical character and its logical structure. These two tendencies which seem diametrically opposed are combined in some poorly understood interaction and create our everyday language in all its diversity. Studying these two tendencies separately and independently from each other will hardly be fruitful.

Language Polymorphism and Gödel's Proof

I consider the papers of the British linguistic philosophical school interesting not so much for the results, and even less for the methods of concrete linguistic analysis elaborated for this purpose, as for certain judgments of a general character concerning language, which we can name, following Gellner (1959),[2] the conception of language *polymorphism*. The term "polymorphism" should obviously be considered more apt than the term "polysemy," widely used in linguistic literature, since here not only words' polysemy is involved but also the general irregularity of language.

The diversity of everyday language is considered its most essential characteristic and is no longer regarded as an index of its deficiency. It is due to its polymorphism that natural language is richer than any artificially created one—in this way the answer of British analytical philosophers to the neopositivists may be formulated. To strengthen this almost self-evident statement, I shall now mention Gödel's famous proof of undecidability. I shall not give here its proof in the strict sense of the word, but only an analogy which seems sufficiently interesting and profound.

Gödel's proof is of an extreme epistemological significance. It completes a whole epoch of profound and unconditional belief in determinism, a belief whose last bright flash was the appearance of neopositivism.

The foundation of the scientific vision is confidence in the necessity of verifying hypotheses. This idea has been formulated especially clearly by the neopositivists, though, of course, it is as old as science itself. Thorough analysis of the logical content of the verification principle, made lately, has shown that, even if we confine ourselves to the natural sciences, we shall see that all is not as well as it should be (for details, see Popper, 1965).

As a matter of fact, according to the Popperian school, verifiability should be replaced by falsifiability: the only thing we can do is to show that a hypothesis formulated by us does not contradict the results of our observations. But if the hypothesis in question cannot be falsified, it does not yet follow that it will be impossible to formulate another, probably even stronger hypothesis which will not contradict observations either. However great the number of observations supporting our hypothesis, they are always insufficient for its unconditional acceptance. At the same time, a single negative result is enough to reject the hypothesis. In view of

[2] The book *Words and Things* by E. Gellner is devoted to a critical analysis of the principal ideas of the British linguistic philosophical school. It is highly readable; the author's criticism is not tiresome and irritating. A critical account of the ideas of the British philosophical school is also presented in the recently published book by Kozlova (1972).

such a troublesome logical asymmetry, we have to introduce a system of conventions formulated in the language of probability theory in order to be able to assess the degree of reliability of our hypotheses [for details, see Chapter II of my earlier book (Nalimov, 1971)]. The problem of verification in mathematics is even more complicated. The heart of mathematical constructions is *mathematical structures*, systems of axioms rich in their logical consequences (Bourbaki, 1948). The verification is here reduced to testing the inner consistency of the structures. The question of inner relations of axioms has troubled mathematicians since ancient times, immediately after the Euclidian axioms, the first mathematical structure well known to us, were formulated. Numerous efforts were wasted in the attempt to infer the fifth postulate from the basic ones. But after non-Euclidian geometries appeared, another problem arose: it was necessary to show their inner consistency. First, mathematicians were satisfied with a *relative* proof of consistency, using a method of mathematical simulation. In the system of old and recognized mathematical structures, models should have been built on which the axioms of new structures could be fulfilled. One system of mathematical constructions was interpreted with the help of another. Thus, it became possible to show that a plane in Riemannian geometry is simulated by the surface of a sphere in three-dimensional Euclidian space, and in this way the postulates of Riemann turned into theorems of Euclidian geometry. Further, it was shown that Euclidian postulates are fulfilled in a certain algebraic model and, consequently, are consistent if algebra is consistent.

The problem of inconsistency in mathematics became especially acute when contradictions were found in Cantor's theory of sets. At the beginning of our century (in 1904), Hilbert, the famous German mathematician, set out to prove the *absolute* consistency of arithmetic, recognizing the insufficiency of relative proofs when one system of mathematical construction is simulated by another. Later, in the 1920s and 1930s, Hilbert and his school published a number of papers in which certain precise results were obtained. These results seemed to prove the consistency not only of arithmetic but also of the theory of sets. But in 1931, Gödel published his remarkable theorem "On Formally Undecidable Propositions in *Principia Mathematica* and Related Systems," whence followed the failure of Hilbert and his school.

Gödel's proof concerns certain logical systems constructed in a certain way. Axioms are there regarded as lines of symbols, and rules of inference are regarded as ways of deriving new lines from lines. Two demands are imposed upon the rules of inference: they should be strictly deterministic and finite. That means that absolutely unambiguous rules are used, and that using them we should not resort to transfinite induction, a method where one would resort to transfinite numbers which appear through generalizing ordinal numbers on infinite sets.

Certainly, I shall not give here the proof of Gödel's theorem (strictly speaking, there are two theorems, but the second one is a mere consequence of the first), as it is too complex. It is preceded by forty-six definitions and several auxiliary theorems. Attempts to give quite simple proofs of Gödel's theorem may be found in Nagel and Newman (1960) and Arbib (1964). Here, I shall mention only briefly that in proving this theorem arithmetization of mathematics is of great importance. Every mathematical statement is encoded with an arithmetical formula. The study of mathematical statements is reduced to studying arithmetical relations.

From Gödel's proof it follows that the generally used consistent systems whereby arithmetic is expressed are incomplete. There exist true statements expressible in the language of this system that cannot be proved within such a system. Further, it follows from the same theorem that it is impossible to prove the consistency of an arithmetical system by use of methods expressible within this logic. From the theorem it also follows that no matter how much expanded the axioms of the logic (the expansion being finite), it will never become complete; there will always be new truths expressible by its means but not inferable from it.

On the basis of Gödel's theorem, certain statements can be made of a generally methodological or even epistemological character. First of all, from the theorem it follows that it is impossible to give a formal definition to the concept of "proof" in mathematics. In the process of the development of mathematics, new methods of proof appeared which were not foreseen earlier. Further, in the above-mentioned book by Nagel and Newman (1960), the impossibility of building thinking machines is asserted since the programs for computers are always composed on the basis of strict logic. General conclusions from Gödel's proof may be formulated as follows: *human thinking is richer than its deductive form.*

We do not know how a human being actually thinks, but we know fairly well that, on the level of communication, formal logic is widely used. In our everyday speech, to say nothing of scientific language, we can easily trace the logical structure (I have already spoken about this earlier). Here, the question arises immediately: What is the mystery of our language? Why does the logical form of communication not submerge any other modes of human thinking which we, perhaps, do not understand but which are obviously much richer? In what way is Gödel's difficulty overcome in our language?

The conception of polymorphism is one answer to these questions. Vague and ambiguous meanings of words, indistinct demarcation lines between words, and the diversity and heterogeneity of words—all of these create the possibility of breaking out of strictly deductive forms of thinking, the breach being polite and not irritating the interlocutor. Human judgments should, on the one hand, be sufficiently logical; i.e., they should be based on deductive logic. On the other hand, they should be built so as to allow breaches in the strict logic of a system of postulates and rules of inference; otherwise, the system will be tautological. Polymorphism of language is a means allowing such breaches to occur without breaking the illusion of strict logic: it allows us to introduce into our system of judgments the inconsistency without which it would be incomplete. This is also true of judgments in the language of mathematics. I shall remind the reader of the following statement from Gödel's proof: ". . . if arithmetic is consistent it is incomplete." Perhaps the same idea has been expressed by the British school of philosophy, which stated that precision and extreme strictness of the language lead to intellectual spasms (Gellner, 1959). Polymorphism of the language allows us to make our *system of communication non-Gödelian* (Nalimov and Mul'chenko, 1972). [After this manuscript had been finished, I became acquainted with the book by Popper (1962) in which he bases his criticism of the logical positivists' conception of an artificial scientific language on Gödel's proof.]

At the same time, we understand that the inner inconsistency of judgments created by language polymorphism should not go too far; otherwise, the situation of an asylum will result. The limit of unstrictness allowed is somehow set by itself. I shall show further that our everyday language occupies an intermediate position in the semantic scale, on one end of which is situated a hard language with precisely determined meanings of symbols, and on the other end, soft languages with an altogether arbitrary relation between a symbol and a referent. Our everyday language does not occupy a strictly fixed place on this scale; it embraces a broad indeterminate area there.

We should not close our eyes to the fact that people have to pay for the polysemy of the language. Odd disputes quite often arise as a result of different interpretations of the meaning of one and the same word, though probably this is an unavoidable component of human creative activity, one we cannot program in an attempt to create an artificial intelligence. The problem of mutual misunderstanding is perhaps most acute in philosophy, which is only natural since there the most complicated ideas are discussed. The British analytical school accused philosophers of using words of everyday language in an unusual sense. And in contrast to the neopositivists, they did not demand correction of language but confined themselves to a more modest program: interpretation and clarification of word meanings and judgments built of these words.

In fact, I believe that a philosopher can construct an interesting model merely by using common words in an uncommon sense. Obviously, this leads to the necessity of interpreting what has been said. Even the well-known *Tractatus* by Wittgenstein needs an interpretation, but all interpretations are poor in one respect or another. If a brilliant thinker tried to present his ideas in such an interpretative manner, they would look too dreary. Certainly, the *Tractatus Logico-Philosophicus* could not have appeared if such style had been required from the author, though the book deals with the rigor of language.

Games including a random component are a peculiar model of language behavior. One such game is cards. When people of different intellectual background meet somewhere—say, in a train compartment—they feel a desire to replace language behavior with its simplified model, a card game. Card games possess strict rules and well-developed strategies applied in random situations. These rules act like the rules of logic in our language; they cannot be broken or you will play not *this game, but another one* (Vendler, 1968). Shuffling cards is here a generator of randomness. Randomness combined with a complex system of rules makes the game intellectually rich, recalling verbal behavior where randomness is given by the language polymorphism. What is important here is that our verbal behavior, as well as a card game, should have several alternatives; otherwise, everything will turn into a farce, into solving a charade, and will be as dull and sad as a trial with a predetermined outcome (Gellner, 1959). Random constituents resting upon a logical structure enter our verbal behavior.

Bayesian Model of Language

Let us now try to build a model of language which will contain overtly the probabilistic structure of the meaningful content of a sign. First of all, I should say a few words about the theorem of the Englishman Bayes[3] and the neobayesian approach to the foundation of the rules of

[3] Thomas Bayes (1702–1761) was a member of the first secure generation of English religious Nonconformists. His father was a respected theologian of dissent; he was also one of the group of six ministers who were the first to be publicly ordained as Nonconformists. Privately educated, Bayes became his father's assistant at the presbytery at Holborn, London; his mature life was spent as minister at the chapel in Tunbridge Wells. Despite his provincial circumstances, he was a wealthy bachelor with many friends. The Royal Society of London elected him a fellow in 1742. He wrote little: "Divine Benevolence" (1731) and "Introduction to the Doctrine of Fluxions" (1736) are the only works to have been published during his lifetime. The latter is a response to Bishop Berkeley's "Analyst," a stinging attack on the logical foundations of Newton's calculus; Bayes's reply was perhaps the soundest retort to Berkeley then available. Bayes is remembered for his brief "Essay towards Solving a Problem in the Doctrines of Chance" (1763), the first attempt to establish firm foundations for statistical inference. (*Dictionary of Scientific Biography*, vol. 1, New York, 1970).

inference in modern mathematical statistics. The principal idea lies in the fact that, making certain decisions after this or that experiment, we always use both the newly received and the previously known information about the phenomena under study. Before an experiment, the experimenter always has some knowledge expressible in probabilistic language; we can call it prior probability or, alternatively, subjective or personal probability. [Ramsey was the first to introduce the concept of personal (subjective) probability in 1926.]

Bayes' theorem allows us to formalize the process of decision making by simulating a procedure where both prior information and that received as a result of an experiment are used; the answer is given in probabilistic terms as a posterior probability.

Let us explain this theorem in ordinary statistical terms. Let us assume that the measurement of a certain attribute μ of a certain object H is effectuated. There is a region Y of all possible results of measurements y. In this region probability $p(y|\mu)$ is given; in the simplest case the errors in the measurement of object H are normally distributed. Further, we shall assume that we know prior probability $p(\mu)$; that is, that a priori before the experiment we know the distribution of all possible values μ. Then Bayes' theorem may be put as follows

$$p(\mu|y) = kp(y|\mu)\, p(\mu)$$

where k is a constant to normalize the result. Prior probability being introduced into consideration gives the entrance into the system, and with the help of Bayes' theorem, we provide a logically faultless exit which is put down as a posterior probability, $p(\mu|y)$. The difficulty of this approach consists in comprehending what prior probability, $p(\mu)$, is. As a matter of fact, this question has been widely analyzed [see, for example, von Wright (1962) and Good (1962)]. In any case, it is clear that a person always estimates probabilities of various events in both his scientific and his everyday activities. These estimates are always subjective in the sense that they are determined by the intellectual disposition and experience of a given subject, and the degree of his being informed; they are at the same time objective to a certain extent, or, it is probably better to say, general since it is assumed that we deal with reasonable observers rather similarly disposed. Another thing is important here: subjective probability of an event estimated by any method may be handled the same as the probability introduced mathematically if it has the same properties and obeys the same axioms.

If a priori we know nothing about the distribution $p(\mu)$ our ignorance may be expressed thus: all values μ are equally distributed on a straight line. In this case it is easily seen that the posterior distribution will be brought down in statistical investigations to the initial distribution of errors in measuring $p(y|\mu)$, built in relation to value $\bar{y}$ found in the experiment. In the long run, we shall compute the estimated value $\bar{y}$ with the same two- or three-sigma limits of traditional statistics. The only difference lies in the logical foundation, but it proves to be very profound. It allows us to avoid the difficulties we have to face in traditional statistics where the confidence limits are given before the measurement is taken; in fact, it is not clear why they should always remain valid after the measurement is taken.

I shall illustrate some logical difficulties arising here with an example. Assume that we are measuring small quantities of a substance in a sample near the limits of detection (Slavnyi, 1969). At first sight it seems natural to regard the value $\bar{y} \geqslant 3\sigma$ (criterion of Kaiser) as a threshold signal (the limit of detection being a standard deviation characterizing the average of background fluctuation). Then we have a simple rule of decision making: the substance in the sample is not found if $\bar{y} \leqslant 3\sigma$. But then we are not using all the information present in the observations. In particular, we do not pay attention to the fact that the value $\bar{y} < 0$ was found for a certain sample though we know beforehand that the content of the substance in a sample cannot be less than zero. Negative values could also serve as a basis for a statement that the true content of the substance in the probe is lower than a certain value significantly less than the 3σ limit of detection. The trouble is that using traditional statistical methods we cannot build a distribution function around the value $\bar{y}$ when $\bar{y} < 3\sigma$ without getting into the region of negative values of a signal, which, of course, lacks a physical sense. In a Bayesian approach, everything becomes much simpler. A prior statement of the impossibility of negative concentration results should be introduced into the decision-making rule; further, all positive values of the signal should be acknowledged to be equally probable (of course, within certain limits). Then at the output of the decision-making system we shall naturally obtain posterior probabilities. In the case of a uniform posterior distribution, the problem will finally be reduced to renormalization: the part of the square under the portion of the curve of the differential distribution function which corresponds to the positive values of the signal will equal 1. This is presented graphically in Fig. 2.[4] Of course, in practical work, it is more rational to take not an equally probable distribution but a decreasing one; however, this is a detail which we shall not dwell upon.

[4] One might ask whether it is necessary to present the results of the substance analysis in such a complicated manner as it is done in Fig. 2. The answer depends here on the problem formulation. If this is the case of a mass quality control, it may not be necessary, but we may come across problems in which we must make a responsible decision on the basis of the results of an analysis carried out in the vicinity of the limits of detection. In this case, a Bayesian solution will seem attractive.

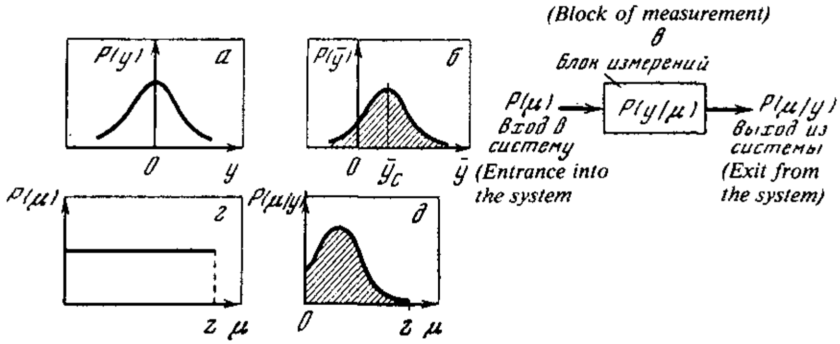

FIG. 2. *Graphic illustration of the application of the Bayesian theorem in the problem of finding small contents of substances.* **Classical solution of the problem:** (a) *the distribution function of the error of measurement known by an experimenter before the performance of the given measurement;* (b) *constructing of 95% confidence limits for the results of measurement by way of centering of the distribution function* **a** *in relation to a new result of measurement* **ye** *(it turns out that we must admit the existence of negative concentration of substance with great probability).* **Bayesian solution of the problem:** (c) *a block-scheme of the Bayesian solution;* (d) *à priori distribution function for the contents of substance in a probe (here the hypothethis of almost complete à priori ignorance is accepted: impossibility of the existence of negative concentrations is the only fact we know);* (e) *à posteriori distribution function p(μ|y), obtained by means of multiplying à priori distribution function* **d** *by the distribution function of the errors of measurement* **b** *obtained by measuring the given probe.*

This example shows how a Bayesian approach allows us to avoid a logically unfounded method of action: ascribing to the observation results the limits of confidence suggested by a statistician before taking measurements. It is especially important that we have managed to build a new procedure for estimating observation results only on the basis of using prior knowledge. A new algorithm of decision making seems quite natural: it is as if it simulates our everyday behavior since in this kind of behavior we make decisions using our prior knowledge and the additional inputs from most recent experience.

At this point, I should probably make a reservation: in the frame of classical statistics it is indubitably possible to overcome logical difficulties connected with defining confidence limits for very small concentrations. But it cannot be done so elegantly as in the Bayesian approach.

Let us now return to the semantic analysis of sign systems. Our principal statement may be formulated as follows: both in everyday language and in many other languages, every sign is connected in a probabilistic manner with a variety of meanings. We may speak of a prior function of distribution of sign meanings. This distribution may be constructed, for example, as follows. The receptor's consciousness contains a certain notion of possible sign meanings: one of them has a greater probability; another, a lesser one; etc. All this may be represented with a distribution function built so that on the abscissa the ranks of meanings are plotted according to their probability of occurrence and on the ordinate probabilities themselves are plotted. The scale of the abscissa may be imagined as continuous: sections of this scale with vague boundaries (such as vague demarcations between colors on the wave scale for the white light spectrum) may be meaningful units. Looking through dictionaries, bilingual or explanatory, we shall see that every entry word is explained by several or many words. These explanatory words are usually ordered according to the strength of their connection with the entry word. Thus, the notion of a distribution function of the word meanings is implicitly present in the structure of our dictionaries. The meaningful content of a sign is given there as a *semantic field* whose elements are ordered on a linear scale. We want to strengthen this arrangement by ascribing to the sections of the semantic scale those probabilities with which they are associated by use of signs. The probabilities appear in the consciousness of the receptor, a subject, and for this reason the corresponding distribution functions may be called prior or, as is sometimes said, subjective or personal.[5] Two examples of such an arranged semantic field are given in Fig. 3. In building distribution functions we have made use of the analysis of the words "game" and "to read" in Wittgenstein's *Philosophical Investigations*, as well as data from the dictionaries of Webster and Dal'.

It is quite obvious that a person with another intellectual background may have quite different distribution functions. This especially refers to the word "game," since it is not difficult to think of a person for whom

[5] There exists a voluminous literature devoted to the techniques of estimating subjective probabilities [see, for example, the paper by Winkler (1967), which has a bibliography of the most important papers]. In elaborating these techniques, the following requirements are taken into account: (i) an expert should *obey the postulates of concordance*, which means that he must express his judgments so that they corresponded to the existing notions of the probability theory; (ii) the expert's judgments must be numerical expressions of his personal opinion. The first of these requirements can easily be tested; the second one cannot, since it deals with purely subjective notions not corresponding to any objective reality, or, to be more correct, to a reality existing outside the expert. Estimating subjective probability is an interesting psychological challenge. If we have a right to state that human behavior is probabilistic, it does not yet follow from this that a person has an ability to express his judgments in the system of probabilistic notions used in mathematical statistics. Real experiments of estimating subjective probabilities are carried out as if a mental lottery is in operation and the expert places bets. Different formulations of the problems often lead to different results. Other difficulties arise as well. For example, it may prove that the expert's sum of probabilities is 1.2. What will he do in this case? He may norm it to a unity dividing it by 1.2, or he may subtract from certain probabilities so that their sum equals a unity. Then, it has become experimentally obvious that naïve experts tend to build truncated distribution functions. They regard the probability of rare events as exactly equal to zero and not as close to zero. In any case, it is clear that experts should be specially instructed. It should also be remembered that probability estimates obtained at various moments may differ significantly, and this must be regarded as experts' natural behavior.

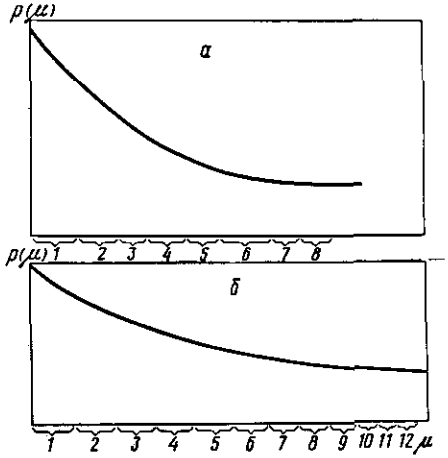

FIG. 3. *Possible à priori (personal) distribution functions of the meaningful content of two words.* (**a**) *The word "game": 1–game as playing situation, a procedure with a variety of nondetermined alternatives; 2–game as a model to describe complex systems (a game model of language, a game model of the world in the Buddhist philosophy, etc.); 3–game as a mathematical model of decision making, mathematical game theory; 4–game as passionate activity (gambling, gambler, etc.); 5–game as competition (in intellectual and political activity, etc.); 6–game as competition in sports; 7–game as joke, enjoyment, mockery; 8–game as theater.* (**b**) *The word "to read": 1–to pass from symbols to speech sounds; 2–to read something learned by heart; 3–to read to oneself, 4–to read proofs, checking the written material by comparing it with another text; 5–to translate unambiguously one sign system into another; 6–to carry out measurements; 7–to interpret in ordinary language something written in other languages; 8–to guess something written in an altogether unfamiliar language; 9–to interpret images; 10–to guess something concealed behind the external manifestation, e.g., to read others' thoughts; 11–to predict, to read one's fate; 12–to deliver lectures (to read a course).*

this word will be primarily associated with gambling games and will in no way be connected with a branch of mathematics.

The prior probability permits the first step in the process of perceiving the text read. The process of reading is used here in a broad sense as a procedure of text perception because the texts are formed of various signs, and we can thus form the $p(y|\mu)$ distribution function. It is given by several factors: by means of combining the sign read with other signs of a phrase or by the general emotional-intellectual disposition of the receptor and by his attention at the moment of reading. The two last factors introduce the same element of uncertainty as an error in ordinary physical measurement. In any case, it seems reasonable to speak of the errors in semantic perception of a sign in the same way that we speak of the errors in any other measuring procedures, as well as to introduce the notion of a distribution function. The analogy can be continued as far as one wishes. Imagine, for example, that you are finding by means of a spectrochemical analysis some element in a sample with a complex structure. The errors of analysis will first of all depend on the general sample composition and on its physical state; further, they will depend on the attention of a laboratory assistant, and on the unavoidable irreproducibility of all elementary measuring procedures, including here the condition of measuring instruments.

At the *exit* from the system, we shall have a posterior distribution function $p(\mu|y)$: after reading, the sign read will still be associated not with one meaning but with a field of meanings whose elements will be arranged in some probabilistic manner. In a particular case of full prior ignorance (or prior indifference), the function $p(\mu)$ will be just a uniform distribution (on a straight line) and then $p(\mu|y)$ will be reduced to $p(y|\mu)$, but this can hardly happen when the receptor is a sane human being. If $p(\mu)$ is more or less similar for the receptor and the transmitter, the process of reading will introduce only casual distortions. But it may turn out that the receptor and transmitter ascribe to the sign system completely different senses. This is evidently what is happening to a certain degree in Western philosophy, and this is what has led to an appearance of a critically minded trend in analytical philosophy which I have already mentioned more than once. This is expressed even more obviously in abstract painting, which I shall dwell upon later. In everyday life, people of one social circle usually meet and they have certain agreement as to the prior distribution functions. But this is not always so. And the more interesting the idea stated, the more striking is the transmitter's prior distribution function connected with the signs used: speaking of something new, he uses old signs. I should like here to recollect one of Wittgenstein's statements, "The silent adjustments to understand colloquial language are enormously complicated" (Wittgenstein, 1955, paradox 4.003).

Different people may read signs differently as well. Divergence of the results of one and the same text being read by different people seems always to be larger than reproducibility during the repeated reading of the same text by the same person. The same happens during physical measurements. Errors of interlaboratory reproducibility are always larger than errors of intralaboratory reproducibility.

It is interesting to note that, using a neobayesian approach in physical investigations, different observers may assume different prior probabilities. This is especially obvious in the problems of discriminating hypotheses, when one has to choose a hypothesis from a number of rival ones.

Inappropriate choice of prior probabilities does not lead here to any grave troubles. Both general theoretical reasons and calculations made when problems are solved by simulating them show that the Bayesian system of decision making has a short memory in the system of sequential procedures: wrongly chosen probabilities are quickly forgotten after several experiments. The same is believed to take place in reading texts. Imagine that the receptor has a prior distribution function different from that of the transmitter. Reading attentively one and the same text, or, better, various texts by the same author or group of authors, the receptor will be able to switch over in the process of reading. This is a process of learning accompanied by forgetting old information. But does it always take place? To provide for this, the receptor should not be too conservative. In any case, prior information is understood here not in the Kantian sense but the way it is commonly understood nowadays in mathematical statistics: in relation to the $(n + 1)$th experiment, prior information will be that received in the nth experiment.

The model described above proceeds from a profound analogy existing between the process of measuring and its interpretation and the process of reading a sign system. It may be contrasted to the well-known concept of *logical atomism* of Frege, Russell, and Wittgenstein (in his early period) (see, for example, Pears, 1956). Logical atoms, elementary and indivisible particles of sense, may be opposed to a continuous distribution function of meanings, and this contrasting goes even farther; I believe that meaning cannot be ascribed to a sign before a text is read, though we have some prior idea of the meaningful field of a sign. This is analogous to the impossibility of ascribing to a value estimated in a physical experiment the confidence limits we believed it to have before the experiment. The analogy with a physical experiment may be continued. If we deal with a continuously changing random variable, the probability of hitting a strictly fixed point in measuring equals zero. A notion of certain unique and strictly fixed meaning of a sign will be a similar non-degenerate case.

Our model can also be contrasted to the statements of Wittgenstein (in his later period). In the *Investigations* (Wittgenstein, 1953) there is his famous phrase that the meaning of a word is given by its use. Some Western philosophers believe it to be the strongest statement of twentieth century philosophy. In our model, the process of word perception is given both by its use expressed as distribution function $p(y|\mu)$ and by the prior knowledge distribution function $p(\mu)$. If the receptor a priori has no meaningful associations with the sign read, then the distribution function is degenerate: the probability of all meaningful associations proves to equal zero, and in this case our model shows perfectly formally that the text cannot be read. This is probably what will happen to messages from other worlds if they are received some day. In his *Investigations* Wittgenstein puts a question: What does it mean that we have begun to understand a text? My answer to this question, contrary to the author of *Investigations*, is as follows: we have succeeded on the basis of our experience to build a prior distribution function of the meaningful content of a sign which previously aroused no associations in our mind.

What is said above may be illustrated with an example from the book

Star Diaries of Ijon Tichy by the Polish writer of science fiction Stanislaw Lem (1971). The reader can find there several words of non-terrestrial origin. Here is how the word "sepulka" is explained in the "Cosmic Encyclopaedia":

> Sepulka – an important element of the civilization of Adrides (v.) on the planet Enteropea. v. sepulkarium.
> Sepulkaria – objects for sepulking (v.).
> Sepulking – activity of Adrides (v.) on the planet Enteropea (v.) v. Sepulka.

Then a dialogue takes place.

> I came up to the counter and with a feigned calm asked for a sepulka.
> "For what sepulkarium?" asked the salesman descending from his peg.
> "Well, for a common one," I said.
> "For a common one?" He was surprised. "But we have only sepulkas with a whistle."
> "Well, I want one."
> "And where is your zhutka?"
> "Ehm, I have not it with me."
> "Where will you take it, then, without a wife?" asked the salesman scrutinizing me and growing dim.
> "I have no wife," I said imprudently.
> "You . . . have . . . no wife?" mumbled the salesman growing black and staring at me thunderstruck. "And you want a sepulka? . . . without a wife . . ." He was trembling all over.

Here we see that an extensive and logically correct text is insufficient to understand the meaning of a foreign word. We have not at our disposal the set of meanings μ upon which we could build the distribution function $p(y|\mu)$. We not only cannot understand, we cannot even feel vaguely, the word-meaning if it has no prior distribution function in our mind. Its logically correct use does not yet reveal its meaning.

If we hold to the Bayesian model of sign perception, we shall have to acknowledge that, although reading a text will cause our consciousness to absorb not a certain discrete meaning connected with a sign read but rather a whole field of meanings, the range will be narrower than the meaning connected with the sign before the text was read. A Bayesian model may be interpreted as a many-valued probabilistic logic; here, the answer is given by a distribution function of sense content.

In the frame of the model suggested here, many facts well known in linguistics are easy to interpret. First of all, we can speak of what lies behind the notion of the term "*precision*" so widely used in discussion of scientific terminology. From our point of view, this very exact notion is determined only by a degree of vagueness of the prior distribution function $p(\mu)$ related to a scientific term μ.

Another well-known problem is that of synonymy, one of the characteristics of language polymorphism. However, it remains unclear what we understand under this term. A statement by Nida (1965) is very interesting in this respect. He denies altogether the existence of synonyms, saying that no morphemes or their combinations are ever identical with respect to the meaning they contain. It is always possible to give an example in which generally acknowledged synonyms proved unequivalent. This is especially readily seen if we turn to set expressions. For example, in the expression "as right as nails" the word "right" cannot be replaced with the word "correct," though these two words are considered synonymous according to a dictionary of synonyms (Aleksandrova, 1971). In our system of ideas, synonymy is given simply by a coefficient of rank correlation between meanings of two different words. We can also estimate a degree of meaningful coordination between several words by means of the coefficient of concordance, well known in non-parametric statistics. It becomes immediately clear whence come the difficulties in defining what synonyms are. In fact, it is doubtful that the rank ordering of the meanings of two different words can be completely identical, and for this reason the phrase by Nida should be understood in the sense that there are no synonyms whose coefficient of rank-order correlation would equal unity. Evidently, we regard as synonyms the words whose rank-order coefficient of meanings is not too small; i.e., speaking the language of mathematical statistics, it should be significant for a certain previously chosen level of significance. Besides, we should not forget that prior distribution functions are subjective, and we should always think of some average sense ranking which characterizes semantic behavior of whole groups of people. Nevertheless, it is reasonable to believe that in reality everything happens the way our model presupposes, and a certain level of significance for the correlation coefficient which is not fixed anywhere is somehow set spontaneously. This allows us to recognize some words as synonyms.

I have attempted to investigate quantitatively the frequency of occurrence of synonymous groups of different sizes in the English and Russian dictionaries of synonyms. The frequency of groups containing two, three, or more synonyms in dictionaries (Webster, 1942; Aleksandrova, 1971) was determined, and the results obtained are graphically presented in Fig. 4. Distribution functions proved to be strikingly different, though, of course, we are not sure that the writers of these dictionaries meant the same level of significance for the correlation coefficient.

The first thing that surprised me was the large difference in the number

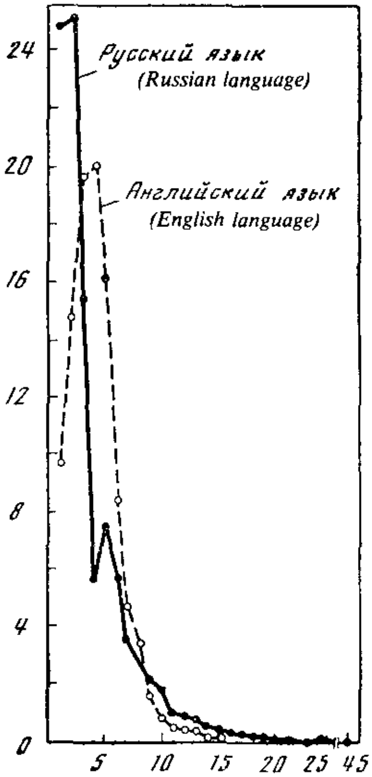

FIG. 4. *Frequencies of occurrence of synonymous groups of different size in Russian and in English. On the abscissas the number of words is marked which enter the synonymous groups; on the ordinates, the frequency of appearance of such groups in the dictionaries of synonyms (Aleksandrova, 1971; Webster, 1942).*

of entries. The Russian dictionary is compiled for 8,322 entries, whereas the English one contains only 1,954. For the latter dictionary an obvious concentration of frequencies is observed for words with a small number of synonyms (from one to five), and then the curve begins to decay rather rapidly; the maximal number of synonyms is 15. The distribution function for the Russian dictionary is extended: there are words with more than 20 synonyms, and the maximal number is 59. Is it a real difference in the semiotic structure of the two languages or just a result of different approaches to the definition of synonyms? In this case, the definition of synonyms in the two dictionaries is formulated so that it is impossible to answer this question, but I believe that statistical study of the synonymy of various languages, if properly organized, could prove very interesting.

A Bayesian model allows us to understand the nature of a joke. Here is an example.

> Recently I made a report in a biochemical laboratory on the way the teaching of mathematical statistics to these specialists should be organized. I began my speech by telling the audience how the for-

mulation of the problem emerged during the talk at dinner in our canteen. After a thorough discussion of the problem, one of those present at the table said, "Now we may come to the conclusion: the service at our canteen should be improved." This remark animated the audience, but a mathematician who was late and had missed the beginning of my report got up and asked with a perplexed air what it meant—probably he misheard my words. However, when it was explained, he began to laugh too.

What has happened here? After my speech, the word "canteen" in the minds of the audience began to be associated (with a very small probability) with the problem of teaching mathematical statistics to biochemists. After the concluding remark, in accordance with the Bayesian theorem, the posterior distribution function connected with the word "canteen" was reduced to a meaning of the word which was associated with it only weakly in the prior distribution function. This shocked and pleasantly animated everybody. The mathematician who had missed the beginning of the report did not have this additional weak association in the prior distribution function; therefore, he did not understand the joke.

A human being is put together so that he does not like dull verbal behavior; it wearies him. A joke breaking the monotony of a speech consists in suddenly transferring associations with small probability into dominant ones. Jokes are based on using the tail part of the prior distribution function. To understand them, one must possess a "stretched tail" part of the prior distribution function. In order to be able to make jokes, one must be able to use this tail part. People who do not understand jokes have a truncated prior distribution function; they can use only those meanings of the word's semantic field which are associated with the word with a large probability.

Many jokes are based on the fact that a phrase contains a word to which two quite different meanings can be ascribed with almost equal probability. As a matter of fact, a word having simultaneously two equally probable meanings is the basis for innumerable puns. The following anecdote based on the present-day Soviet reality is an illustration of this:

> One nationalistic Jew says to another during the six-day war, "I say, yesterday our soldiers shot down six of our aircraft."

The gist of this anecdote is based on the two meanings of the word "our"; in the first case it signifies Soviet soldiers and in the second one Israeli aircraft.[6]

[6] In Russian the anecdote sounds better since the Russian pronoun "our" (наши) in the first meaning is substantial; i.e., it does not require a noun.

Now let us see in what way we can interpret this joke in Bayesian terms. It seems that upon hearing this phrase I recall my own prior distribution function of the word "our." If I try to ascribe to the meanings of this Russian word the probabilities with which they occur in my mind and rank them, I'll get roughly the following sequence: kith and kin, my friends, my colleagues and members of our invisible college, my distant acquaintances, my compatriots, people whose native tongue is Russian. Somewhere at the end of the list, the word "наши" with a very small probability will acquire the meaning of the national minority from which my father comes.

In order to get the gist of the joke, it is very important to remember the remark that this is a conversation of two nationalistic Jews. Taking this into account, the distribution function $p(y|\mu)$ should be constructed proceeding from the peculiarities of nationalistic psychology. In this case the greatest probability among the meanings of the word "наши" is ascribed to belonging to one's nation. The probability of all the other meanings decreases sharply. The product of the two functions—the prior distribution function $p(\mu)$ constructed above and the function $p(y|\mu)$ constructed while reading the phrase and its context—gives a bimodal distribution function $p(\mu|y)$. The word "наши" thus may have two essentially different meanings with almost equal probability, and the phrase may be understood in two different ways. Here lies its pungency.

A brilliant example of a pun based on the two meanings of the word "to shoot"—(i) to aim and fire and (ii) to photograph—may be found in Evelyn Waugh's novel *Vile Bodies*.

> As Adam walked up the drive two lorries thundered past him. Then a man appeared with a red flag.
>
> "Hi! You can't go that way. They're shooting in front . . ."
>
> Wondering vaguely what kind of sport this could be, Adam followed the side path indicated. He listened for sounds of firing but heard nothing. . . . He had not gone very far in his detour before he was again stopped . . .
>
> "Here, what in hell do *you* want?" said the Bishop.
>
> "I came to see Colonel Blount."
>
> "Well, you can't, son. They are just shooting him now."
>
> "Good heavens, what for?"
>
> "Oh, nothing important. He's just one of the Wesleyans, you know—we're trying to polish off the whole crowd this afternoon, while the weather is good."
>
> Adam found himself speechless before this cold-blooded bigotry.
>
> "I dare say you'd like to come round to the front and see the fun," continued the Bishop. "I should think they'd be just singing their last hymn now. It's been uphill work," he confided, "and there's been some damned bad management. Why, yesterday, they kept Miss La

Touche waiting the whole afternoon, and then the light was so bad when they did shoot her that they made a complete mess of her—we had the machine out and ran over all the bits carefully last night after dinner—you never saw such rotten little scraps—quite unrecognizable half of them. We didn't dare show them to her husband—he'd be sick to death about it—so we just cut out a few shots to keep and threw away the rest. I say, you are not feeling queer, are you? You look all green suddenly."

"Was—was she a Wesleyan too?"

"My dear boy, she's playing lead. . . . She's Selina, Countess of Huntingdon. . . . There, now you can see them at work."

The pun of the dialogue proceeds from the fact that Adam's partner speaks so that his interlocutor chooses the meaning of the word "to shoot" which is irrelevant for the situation.

Here is an example illustrating how misunderstanding of the second meaning of a word can be turned into a joke.

Once in a shop I was a witness to the following conversation.
"Have you any cinnamon?"
"We don't sell it."
"And where can I get it?"
"Here."
"But you say you don't sell it."
" 'Don't sell it' means 'sell it only very rarely.' "

Here, the salesgirl explained to the customer a Bayesian model of understanding the two meanings of the phrase "we don't sell it."

A Bayesian model may help to explain the mechanism of comprehending "contracted" phrases. And the meaning of such phrases becomes clear not because of their grammatical structure, but through the Bayesian procedure of passing from a broad meaningful subset of word meanings to one of its subsets. This latter becomes the field of elementary events on which the probability of comprehending the word meaning in a given context begins to be redistributed. It is noteworthy that the mature English language with its highly expanded polymorphism contains such contracted phrases much more often than the comparatively young Russian language in which the tendency to build expanded phrases prevailed up to recent times. By the way, here lies the origin of the well-known fact that after translating an English text into Russian it always becomes slightly longer (in the number of printing symbols). However, this problem is to be studied separately.

As an illustration of the differences in the two languages, I shall use the word combination "New English Bible." Its meaning is quite clear for English-speaking people. Their prior distribution function for the word "Bible" is such that its being combined with the word "new" may mean only a new version of translating the Bible. But the Russian reader, even with some knowledge of English, is puzzled by the title. According to the tradition of Russian lexicographers, the meaning of this phrase should be interpreted, "This is a new Bible, the English one," but such an interpretation contradicts the prior meaning of the word "Bible." Thus, the title of the book should have been translated into Russian as "New English Translation of the Bible."

The English language, despite its strict grammar, allows formally directed phrases. For example, phrases like "It's the last tram but one; the last one will go by in an hour" have become linguistic clichés. If the word "last" is ascribed a discrete meaning, the phrase will have to be acknowledged as logically illicit, since then the existence of idioms would be impossible. But if we believe the word to have a fuzzy meaning including not only the latest event in a time sequence but also an event close to the end of the sequence, then it becomes clear that the word combination "but one" allows us to select from the fuzzy semantic field of the word "last" its subset of meanings pertinent here.

The probabilistic model of language can explain certain peculiarities of the verbal behavior of insane people (my attention was drawn to this fact by psychophysiologist I. M. Feigenberg). It may be suggested that schizophrenics have much flatter slopes of the prior distribution functions of meaningful word contents than normal people, and sometimes the slopes may go almost parallel to the horizontal axis. In any case, the values of word content which with normal people are situated in the tail part of the distribution function arise in the insane consciousness with a probability equal to that of the principal meaningful components of the word. It is not logic but rather semantics of speech which is broken here. Say a patient is asked, "Are there any common features between a plum and a river?" He answers, "There are stones both in a plum and in a river." When asked, "Are there any common features between gasoline and a symphony?" he answers, "If you make a hole in the gasoline can, the gasoline will run out and produce a melody." These answers are not in the least humorous. Patients with such verbal behavior cannot perceive humor, which, as we have already pointed out, is connected with unexpected use of a word's meaningful content. With such patients, the prior distribution function of word content is arranged so that there can be no unexpectedness.

According to I. M. Feigenberg, such psychic disturbances are the breach of probabilistic ordering in the memory "card index." In psychiatry a very curious, though rare, phenomenon is known—that of a "second life" of a patient. Having recovered, the patient returns to normal life but he chooses a new, intellectually simpler profession. He turns out to have maintained his previous knowledge; it is not forgotten but is probabilistically disordered. Here, it must be pointed out that the formal construction of the speech of this kind of patients remains logically strict. Speech disturbances are not of a logical but a semantic nature.

In his book Feigenberg (1972) develops a very interesting conception of probabilistic expectation of human behavior. Imagine, for example, that you are at the railway station, where the broadcasting center gives out quite incomprehensible messages. Still, you will immediately recognize the number of your train. This is Bayesian recognition. This is also true of listening to broadcasts in one's native and in foreign languages: in the former case we get the meaning even if the message is accompanied by noise, while the understanding of foreign texts requires a very high quality of program listenability. Our recognition in this case is hampered since in our consciousness there does not flash a set of alternative words (they must appear very quickly so as to match the tempo of the program), one of which we must recognize. When tuning the radio, you come across some foreign program, and you first of all wish to guess what it is about. As soon as you succeed, the Bayesian mechanism of recognition starts working.

Probabilistic expectation is well illustrated by the well-known Charpentier experiment. Imagine two objects of equal weight but essentially different volume which seem to be made of the same material. A normal person, having picked up the object, will immediately state that the smaller one is heavier. This is a shock response since, judging by their appearance, he expected the smaller object to be lighter. If the same procedure is carried out with closed eyes and the objects are picked up by the string fastened to them, the weights will be perceived as equal; in this case, probabilistic expectation is absent. Schizophrenics do not possess it either: they will not state that the smaller object is heavier even if they see the objects before taking them into their hands.

Now let us pass to discussing the most interesting problem—that of constructing a system of logical judgments by using a set of signs of a polymorphous language. Is it at all possible to build any logical structure if words have a variety of meanings?

A statistician, when formulating judgments on random variables, gives a distribution function by their parameters: mathematical expectation, dispersion, asymmetry, excess. Not all of these parameters are necessary; often it is enough to deal with one of them, mathematical expectation implying the mean of a random value. In our verbal behavior in the process of constructing logical structures, we replace distribution functions with average meanings which are signified by this or that word. An average word meaning is its semantic invariant with a poor sense content. Logical structures may be constructed on various levels of abstraction. On the low levels, the average meaning is very important. On the highest levels, we operate with words simply as with signs, forgetting about their average meanings. In such an abstract phrase, words play the role of logical variables. Thinking over the statement given by this phrase, we try to interpret its meaning by means of words, turning to the mechanism given by a prior distribution function and Bayes' theorem. Strictly speaking, logic can deal only with signs and not with sense. Here, I should like to recall the well-known statement by Wittgenstein (1955) in his *Tractatus*:

> In logical syntax, the meaning of a sign ought never to play a role; it must admit of being established without use being thereby made of the *meaning* of a sign . . . (from paradox 3.33)

Cherry (1957) illustrates this thought by the following syllogism:

> All hoodles are snurds.
> This gabooge is a hoodle.
> Therefore it is a snurd.

This is an example of a deductive inference that is clear for us though built from meaningless words (we have no prior distribution function of sense content for them). We perceive the words of the syllogism as abstract symbols, being quite aware that the moment we get the key to their understanding the syllogism will immediately be interpreted.

The progress of logical thinking is related to passing to still more and more profound symbolization which we generally call abstraction. The language of complex statements becomes two-staged. On one stage, logical statements about abstract symbols are constructed. On the other stage, these symbols are interpreted. The two-staged language structure is especially vividly seen in describing physical phenomena in the language of mathematics. We have already spoken about the profound links between logic and mathematics. In principle, any logical statement may be expressed in mathematical language, and any mathematical statement may be interpreted as logical, but, of course, it is very difficult to do this in practice. Symbols used to fulfill this task have a definite physical sense but not so precise a one as the neopositivists used to demand. At an early stage of development, the physical meaning of symbols played a great role in the process of building a mathematical theory of physical phenomena. But gradually, as the progress of physical knowledge continued, the abstraction of conceptions became more profound. After certain mathematically expressed logical operations, sometimes quite complicated, are defined in terms of relations constructed from abstract symbols, we obtain new relations to be interpreted in the language of experiment.

A very interesting question may be formulated which, for some reason or other, has not been discussed by philosophers: What is the difference between the modern tendency toward mathematization of knowledge and the program of the neopositivists? Both seem to concern the same problem: transforming sciences into formal calculi. Two very different approaches are suggested for the solution of this problem. The neopositivists wanted to construct a calculus on the terms of a new language with strictly unambiguous word meanings. The modern tendencies toward mathematization of knowledge are directed at constructing a calculus over symbols whose meaning does not seem very pertinent. Certain formalization of knowledge is allowed, the set structure of scientific language being preserved. Strictly speaking, two languages are introduced in this case. One of them is a language of mathematics by means of which the system of inference over abstract symbols is built. Another one is an everyday polymorphous language of science by means of which statements received in a symbolic form are informally interpreted.

Now let us consider the problem of translation. Assume that we have to translate a text from μ-language into η-language. From experience it is known that each meaningful word of μ-language may be represented by several seemingly equivalent words in η-language. This means that the word μ_i in question has a field of meanings that completely, or at least in its major part, intersects with the fields of a certain $\varkappa$-subset of $\eta_1, \eta_2, \ldots \eta_\varkappa$ words of η-language. Hence, the following recommendation for translation would seem rational: choose the word from the $\varkappa$-subset which has the greatest rank correlation coefficient with the word μ_i. However, this recommendation is not correct. It might turn out that the phrase in μ-language may be built so that the word makes use of the tail part of the content distribution function which is absent in the words of the $\varkappa$-subset of η-language. To make sure of that, we have to use the procedure of Bayesian reading. Thus, two possibilities emerge. The first is to select a word in η-language into which the meaningful content (understood for μ-language in the Bayesian way) would enter with the maximal probability (this word may prove outside the $\varkappa$-subset). Such a translation will be but a clumsy paraphrase of the text. The second way is to construct in η-language a phrase so that in order to convey the sense expressed in μ-language one will also use only the tail part of the content of a certain η-word. This will be a translation reflecting not only the meaning of the phrase but its mode of expression as well. Now imagine the problem of a dialogue between a human being and a computer. The logic of the conversation must be so arranged that the computer would understand the subtlety of human speech and then start translating the text into some clumsy language, "thinking" in that language, and answering in it. This is a model of our conversation with foreigners: speaking with us, they use the whole richness of their language, but answering them in their language, we use only primitive phrases. Hence comes the paradox: it is easier to speak the foreign language than to understand it; comprehension requires the mastery of all of the most complicated aspects of language. Technically, it seems easier to create programs for a dialogue between a human being and a computer than for a good translation.

From the above-developed standpoint, it seems very interesting to consider the structure of jargon languages. One of them is the "filthy" or obscene language. In some microcollectives in the USSR, it is used every day in place of our usual language. This is an extremely interesting phenomenon of Russian culture, and, of course, it is worth a serious study. Since similar social phenomena occur in other countries (though they have different linguistic features), I shall confine myself to separate statements to the point.

The words of the "filthy" language are devoid of the selective prior distribution function. Strictly speaking, they mean nothing or everything. Their direct meaning has nothing to do with the statements built from them: it only adds a saucy flavor to speech. But if this saucy flavor is viewed as negligible (which is really so from the point of view of semantics), then the filthy words can be replaced by any other words-symbols with equal probability signifying anything one wishes.

Naturally, a conversation in such a language has very poor semantics. People speaking in the filthy language do not use all their rich information about the world encoded in the prior distribution functions of the semantics of the words of everyday language. Language games, in their usual sense, prove impossible in the filthy language.

But where lies its attractiveness? In no way is it in the saucy flavor of the statements. It turns out that the filthy language is fit for fairly special and very attractive language games. At least two constituents of such games can be indicated. The first one is guessing the sense the interlocutor wants to express, in the given situation, in the words of the filthy language. The second one—and this seems especially interesting— is an amazing possibility of creating words: formation of new, quite unexpected words from one root, the initial obscenity.

This question is worth a more detailed discussion. Word formation by means of broad use of suffixes and prefixes is one of the features of the Russian language that makes it very rich. In Russian we can derive from one word, by means of affixes, at least five or six other words of the same grammatical class, which is utterly impossible in English. In French such derivation is possible only to a small degree. (For some reason or other, in his comparison of Russian and English Nabokov did not pay attention to this peculiarity; this comparison is mentioned in the discussion of mathematics as the language of physics in Chapter 4.) This amazing peculiarity of the Russian language was especially vividly revealed by the outstanding Russian poet Velemir Khlebnikov. One of his most famous poems illustrating the versatile ramifications of the derivatives of the word "laughter" is as follows:

О, рассмейтесь, смехачи!
О, засмейтесь, смехачи!
Что смеются смехами, что смеянствуют смеяльно,
О, засмейтесь усмеяльно!
О, рассмешищ надсмеяльных - смех усмейных смехачей!
О, иссмейся рассмеяльно, смех надсмейных смеячей!
Смейево, смейево,
Усмей, осмей, смешики, смешики,
Смеюнчики, смеюнчики,
О, рассмейтесь, смехачи!
О, засмейтесь, смехачи!

What is this? This is a dance of words, in which they are bending, serpentining, wrapping themselves in a cover spun from suffixes and prefixes; they change their clothes before our eyes and turn naked again. The rhythm of this dance-masquerade carries away with it semantic fields of such simple words as "laughter," "to laugh," "burst out laughing," etc. But the principal thing is not the meaning; it does not become clearer due to this masquerade. The principal thing is rhythm and it is utterly untranslatable into any other language, including even Polish despite its genetic relation to Russian.

This poem is based on the single Russian root "смех," which means laughter. Almost all of the derivatives used here do not exist in Russian and are not registered in any dictionary. The thing is that these words were created by Khlebnikov. They are neologisms formed according to the laws of the Russian language. Since they are made by using affixes with whose meaning we are acquainted from our verbal experience, we can get the meaning of each word, though we have not come across them before and will never meet them again. However, their meaning is understood only vaguely, and when we try to render it in English, a language with different means of word derivation, the result is a monster of a poem:

O, burst out laughing, laughing creatures!
O, start laughing, laughing creatures!
They who laugh with laughters, who are engaged in laughter laughingly,
O, start laughing, rocking with laughter
O, of mocking roar of laughter – laughter of risible laughing creatures
O, die with laughter laughingly, laughter of mocking creatures
Laugh 'em to death, laugh over, little laughters, little laughters,

Teeny laughters, teeny laughters,
O, burst out laughing, laughing creatures!
O, start laughing, laughing creatures!

In order to render the poem in English we have had to use about three dozens of extra words with roots other than "laughter." The words "смехачи" and "смеячи," which translate into English in the same way, as well as "надсмеяльные" and "надсмейные," which are translated as "mocking," are morphologically quite different words in Russian.

Now it seems pertinent to ask: Where does the Russian language realize its rich possibilities? The answer is sad. If this potential is realized anywhere, it is, first of all, in the filthy language with its unusually versatile and almost arbitrary word formations. Not wishing to join the ranks of authors writing obscenities, I won't give an example. I am sure that any Russian can easily reproduce them, guided by his imagination or resorting to folklore or even works of the well-known Russian writers; however, it is unlikely that anybody remembers now such once widely known poems by Yesenin as "Большой матерный загиб" and "Малый матерный загиб" (these are the poems which are mostly written in obscenities and their derivatives). One thing I wish to emphasize here is that there cannot be an analogue to the Russian profanities in English, since the latter lacks this specific faculty for creating short-lived words (ephemerides).

It is a pity that Wittgenstein's proposition that a word's meaning is given by usage can be referred to the utmost degree only to such jargon languages as the one described above.

In any case, I consider the questions touched upon worth a most serious analysis. Their study allows us to understand peculiarities of our language. Should we avoid them as a result of false shamefacedness? We know that only that which we have agreed to consider as such becomes obscene.

Concluding this section, I should like to note that our model of language [first briefly formulated during our study of the language of abstract painting (Andrukovich et al., 1971)] is a completion and development of a widely accepted English-American linguistic literature model of a "dipper" (Laird, 1961). According to this model, we may speak first of all of the concept of reference. A word is referred to a definite object or several objects. This property of words is defined more or less precisely. Reference creates but a poor language; people go farther and ascribe a particular meaning to a word. It is stated that the meaning of words is "dipped" from the human consciousness. A word is a "dipper" common to everybody, but the content drawn by the dipper is far from being the same for different people. Laird (1961) gives the following example: Imagine that a man is going to the theater, and his woman companion tells him, "Wait a minute." In this phrase the word "minute" is very far from the astronomical concept of a minute. Depending on the circumstances and character of the companion, this word may at one time mean that it is no use hurrying now—you will have to wait for a long time; next time it may mean that in fact everybody is ready to leave home in a minute.

I would say that referring a word to a certain meaning is equivalent to our notion of its average meaning, and for this reason it unavoidably proves poor. Dipping profound content is equivalent to our notion of a mechanism of Bayesian reading. It is no use looking for some unconditional and unambiguous meaning in phrases. According to Hutten (1956) ". . . it is better to speak of sentences *being meaningful* rather than having meaning . . ." This elegant formulation is supported by our model.

I would like also to oppose our approach to that of the Swedish school of lexicostatistics formed in the 1950s and to its new ramification presented quite recently by Sankoff (1969). The school seems to proceed from the above-mentioned neopositivistic concept of logical atomism. In any case, the conception developed by them is based upon the postulate of the existence of a certain set of meanings. In the general case, this is just an analytical construct. In the particular case, in studying natural languages, this set may be built empirically, proceeding from the analysis of word frequency curves. The original formulation of the Swedish school even stated that it is possible to select 200 principal universal meanings independent of the peculiarities of a given culture, although Sankoff does not make use of this postulate. Further, a stochastic process is considered giving fluctuations and probabilities of using words to express this or that meaning. This is a diffuse process with a zero shift; in the simplest case this is a Brownian process.

Thus, we see that two approaches, apparently rather similar, are possible in constructing a probabilistic model of language, though they lead to essentially different constructions. According to our approach, a distribution function for the word meaning is built, which allows us to use the Bayesian theorem, introduce the notion of subjective probabilities, and obtain all the results described above relating to understanding the way the reader perceives the text. According to the Swedish school, the word fluctuation around logical atoms is studied, which allows one to comprehend certain lexico-statistical phenomena, e.g., the Zipf law well known to linguists which characterizes word distribution on the basis of frequency of occurrence in texts.

Our concept perhaps comes closer to a non-probabilistic approach to building quantitative semantics of the "fuzzy" meaning of words which is being developed by Zadeh (1971) on the basis of his concept of fuzzy sets of the logic of slipshod predicates. But in this system, as far as I can gather, all the attempts to build a communication model which would reflect the peculiarities of semantic disposition of the recipient ended in failure.

The Role of Contradictory Statements

Strange as it may seem, very little can be said about contradictory statements. As far as I know, nobody has made a systematic study of the problem from a generally linguistic stance, though it seems obvious that there is an object of study here: human thinking, contradictory by its nature, should reflect this characteristic in language.

Wittgenstein, at least in his early period, indubitably believed in the logical structure of language and thought that logically contradictory statements simply cannot exist. Here is one of his formulations from the *Tractatus*.

> To present in language anything which "contradicts logic" is as impossible as in geometry to present by its co-ordinates a figure which contradicts the laws of space; or to give the co-ordinates of a point which does not exist (paradox 3.032)

And further: "Most propositions and questions, that have been written about philosophical matters, are not false but senseless." He gives the following example of such a senseless phrase:

> . . . whether the Good is more or less identical than the Beautiful (from paradox 4.003)

Later, in his *Investigations* (Wittgenstein, 1953), he wrote:

> 500. When a sentence is called senseless, is not as it were its sense, that is senseless. But a combination of words is being excluded from the language, withdrawn from circulation.

At the same time logicians (and Wittgenstein also was a logician) from the times of ancient Greece did their best to formulate logical and semantic paradoxes. Many of the latter were perceived tragically like serious contradictions in theories nowadays. For example, it is known that Frege during the last twenty years of his life did not publish a single important paper on logic. This was due to Russell's finding an unsolvable contradiction in one of his publications.

However, this is true of gross irritating contradictions. Many of them, probably all of them, are removed under close scrutiny if we are bold enough to reject a too narrow comprehension of formal logic. I have already remarked that many contradictions arise only because of the heterogeneity of our language: in everyday language we are mixing judgments made in the object-language with those made in metalanguage. Other contradictions result from ascribing to words too precise meanings. These gross contradictions will disappear as soon as we make use of the probabilistic model of language. Let us consider the classical paradox of a liar.[7] It goes as follows: "A liar can confess he is a liar. In this case he will tell the truth. But a person speaking the truth is not a liar; thus it is possible that a liar is not a liar." From the probabilistic viewpoint there is no paradox here at all. The thing is that a person who makes truthful statements with a small probability will be called a liar. The paradox will also stop being such if we analyze it in the following manner: the liar's confession that he is lying should be regarded as a metastatement. Sometimes the paradox is formulated as follows: "The Cretan Epimenides said, 'All Cretans are liars.' However, Epimenides himself is a Cretan and then he has told a lie. What, then, will be the true meaning of his statement?"

Here it is quite easily seen how the statements made in object-language and metalanguage are mixed. If the demand of homogeneity was laid upon the language, Epimenides, being a Cretan, would not have any right to make any judgments of the truth or falsity of their statements.

Now imagine the following situation: a person says, "I am lying." What does that mean? If he has just been speaking about something else, e.g., proving a theorem or making calculations, we understand that in this phrase he informs us that he himself has hit a mistake in his logical reasoning or calculations. In this case the polymorphous word "to lie" realizes one of its meanings. But if it was only the above-mentioned phrase which had been spoken and nothing more, then it just cannot be interpreted. From a formally logical standpoint this grammatically correct phrase conceals an inner contradiction: it is unclear whether the person speaking tells the truth or a lie. This is a stumbling block for logicians. But from our viewpoint there is no contradiction here: this is just a good example of behavior of the probability model of language. We proceed from the fact that the verb "to lie" is a highly polymorphous word. If a phrase contains but two words and has no verbal surrounding, we cannot cope with the polymorphism of the word and have to ascribe to it one strictly fixed and unconditional meaning; one might note something analogous in the process of ascribing a zero probability to the point value of the results of a continuous random variable. This unavoidably leads to absurdity. It is also possible to interpret the above-mentioned phrase in a

[7] Curious legends are associated with the paradox, showing how tragically it was perceived. Diodorus Cronos, a Greek philosopher, died of distress seeing that he could not solve it; a certain philosopher Philip Kossky committed suicide; Chrysippus, a stoic philosopher, devoted three books to the paradox (Kondakov, 1971).

broad sense assuming that the person speaking is telling the truth; namely, that usually, or more often than not, he lies. In such an interpretation the phrase immediately acquires the meaning coordinated with a probabilistic notion of the meaning of the word "to lie." But this interpretation is hardly correct.

The problem of the truth so much investigated in modern logic (see, for example, "Logical Truth" in volume 3 of *Soviet Philosophical Encyclopaedia*) loses its original sense if we ascribe a field of meaning to the words from which statements are formulated.

The probabilistic model of language allows us to understand the way fine contradictions are introduced into speech; they enrich speech and turn it into a non-Gödelian system. This primarily occurs on the level of interpretation when, in reading the words of a phrase, the reader puts into them prior information resulting in a situation of logical conflict and in a collision of ideas. From a formal standpoint this might annoy and even be perceived as nonsense. We shall refer here to an interesting paper by Danoyan (1970), who analyzed contradictions and tautologies in the scientific language of psycho-physiological problems. Danoyan regards as contradictions a binary combination of notions opposite in meaning and being in the formal relation of inclusion. These opposite notions are not mere antonyms but complementary components of a classical dilemma. He states that from the dilemma "physical–psychical" it follows that the phrase "the physical of the psychical" is contradictory; from the dilemma "mechanism–objective" it follows that "mechanism of the objective" is contradictory, etc. Then the following statements are all contradictory:

1. We do not know the physical basis of thinking.
2. What is the physical basis of memory?
3. The physical basis of recognition has not yet been studied.

The contradictory phrase "the physical of the psychical" is the invariant of all these statements. The following statements so frequent in our modern language are, according to Danoyan, of the same type.

1. We do not know the mechanism of thinking.
2. What is the mechanism of recognition?
3. The mechanism of recall has not yet been studied.

The word "mechanism" is here interpreted as "an instrument," i.e., a means of achieving an objective, where the objective is thinking, recognition, and recall. Thus, the contradictory statements "the mechanism of an objective" or "the means of an objective" are an invariant of the above phrases.

I believe that, were this analysis continued, contradictions of this kind might be found in any scientific paper or any discussion. But how we should impoverish our language if we regarded such statements as inadmissible!

It is interesting to note that our language possesses a special mechanism for introducing fine contradictions into it. This mechanism is the use of metaphors. This term cannot be given a good definition. The word "metaphor" comes from the Greek word μεταφορα—a transfer, in a figurative meaning. The *Oxford English Dictionary* gives the following definition of metaphor: "The figure of speech in which a name or descriptive term is transferred to some object different from, but analogous to, that to which it is properly applicable." According to an apt remark by Barfield, when we use a metaphor, we are "saying one thing and meaning another" (cited from Black, 1962). In our interpretation, speech containing a metaphor is constructed in the following manner: a word with a broad prior distribution function of the meaningful content is introduced into a phrase, a part of the word's meaningful content being in accordance with the other words of a phrase, and the rest of it contradicting them. In this way a fine contradiction is introduced into the speech, making it elegant and even, sometimes, refined.

In speech rich with metaphors, the transfer of meaning takes place on the basis not only of similarity but also of contrast. Words are used in a new and sometimes shocking sense; for example, the Russian word combination "сапоги всмятку" has the meaning "soft-boiled boots." Metaphorical speech is primarily associated with poetry, as in the phrases "murmur of the waves," "stacks of the sun," or the following lines from Yesenin's poems (my word-for-word translation):

Что ж ты смотришь так синими брызгами? . . .
Why are you looking with your blue sprays? . . .

Все равно любимая отцветет черемухой . . .
My beloved one will shed her bird-cherry blossoms . . .

We cannot formulate criteria that would enable us to tell a metaphor from a poetic image. For example, are the following lines from a poem by Yesenin metaphorical?

Словно я весенней гулкой ранью
Проскакал на розовом коне . . .
As if I rode a rose-colored steed
In the spring hollow early hours . . .

Не жалею, не зову, не плачу,
Все пройдет, как с белых яблонь дым . . .
I am not sorry, not calling, not crying,
All will pass as white apple-tree haze . . .

Another thing is important to note: our everyday, business and scientific speech are all filled with metaphors. Here are several examples from business speech:

A fashionable branch of knowledge
Intellectual field
Statements orthogonal to
To debug = to eliminate errors

The last is a word of American business slang that has the literal meaning "to squash a bug." The word "bug" has other metaphorical meanings: a technical blemish, a mad idea, insanity, clandestine listening device.

In the chapter devoted to the language of science, we shall return to the analysis of metaphors. To conclude this brief discussion of metaphors, let us note only that the metaphors listed above contain the same contradictions as the examples from the paper by Danoyan. In the first, knowledge is something serious and quite opposite to what is called "fashion." Intellect is something compact, purposeful, and logical—quite the opposite of what we think of as a field, i.e., something wide, fuzzy, and most surely lacking intellect. Our notion of statements is not in any way coordinated with that of vectors, though we know that if vectors are orthogonal that means they are linearly independent, and we understand that the statement in question does not coordinate with some others made earlier. Debugging is a process slightly similar to correcting a mistake but simultaneously quite different from it. Can we not infer, then, that metaphors are not a special category of our everyday language but rather a most vivid manifestation of the mechanism more or less inherent to any speech. Precisely for this reason it is difficult to formulate accurate criteria for defining and selecting metaphors—there is no clear-cut line between common and metaphorical word usage. Metaphor is probably most vividly manifested in oral speech where it is accompanied with additional expressive means: articulation, pitch, and sometimes gestures. Black (1962) gives the following example: When Churchill in his famous phrase called Mussolini "that utensil," it was the tone of voice, the verbal setting, the historical background which helped the English people understand the meaning of the metaphor. From a formally logical standpoint, using metaphors is a rejection of one of the basic laws of logic, the law which may be put down as follows: A either is B or is not B.

Now let us see in what way science reacts to fine contradictions. I have already mentioned that Carnap suggested rejecting *Tractatus* by Wittgenstein as a work full of nonsense. However, nobody has done it and there is hardly anybody who would deny the remarkable intellectual power of this book, though many of its statements can give rise to strong objections. The power of the work is in fact due to its paradoxical nature; separate statements in a certain sense contradict one another though they possess a certain inner consistency, too. It is only through this game of consistency and contradiction that Wittgenstein managed to express elegantly a very complicated outlook which could hardly have been expressed in strictly deductive and inwardly consistent statements.

Here, it seems pertinent to draw the reader's attention to a very interesting though not well-known publication by the physicists Podgoretskii and Smorodinskii (1969) on axiomatic construction of physical theories. According to their veiwpoint, the creation of physical theories goes through two stages: (i) local theories with hidden contradictions, and (ii) revealing contradictions when local hypotheses meet. Overcoming these contradictions proves to be a starting point for the further development of physics. We would like to find out what language means are used in the first stage, in creating local theories. The contradictions should be well concealed at the beginning; otherwise, the paper could not have been published at all.

Even at the stage of completion, in constructing concepts generalizing a macroworld, we have to allow contradictions to arise. Classical logic proves insufficient for the description of the outer world. Trying to comprehend this philosophically, Bohr formulated his famous *principle of complementarity*, according to which in order to reproduce an integral phenomenon in a sign system, mutually exclusive *complementary* notions must necessarily be used. This requirement is equivalent to expanding the logical structure of the language of physics. Bohr uses a seemingly simple means: mutually exclusive use of two languages, each of which, being based upon common logic, is recognized as permissible. Such languages may describe physical phenomena that exclude one another, e.g., continuity and atomism of light phenomena, etc. Sometimes the principle of complementarity is regarded as a generalized principle by Heisenberg. Bohr himself was well aware of the general philosophical character of his principle (Bohr, 1958a):

> . . . Living organisms and the characteristics of people having consciousness as well as human cultures themselves possess the traits and qualities whose depiction demands using a typically complementary way of description.

In our philosophical literature, the principle was at first met with a great caution, but, according to the papers of a conference in Obninsk (*Printsip Dopolnitel'nosti i Materialisticheskaya Dialektika,* 1972), it has lately attracted attention and has begun to be interpreted broadly not only in conformity with physics but with other branches of knowledge as well. The principle of complementarity is, as a matter of fact, an acknowledgment of the fact that theories built in accordance with precise logic work as a metaphor: they become the basis of models, acting as if they were the outer world but not quite so. One logical structure is not enough to describe the whole complexity of a microworld. The peculiar philosophical significance of quantum mechanics consists in the fact that a demand to violate conventional logic in constructing the image of the world first made itself explicit here. It is interesting to cite here a statement by Heisenberg (1958): "Absolute fulfillment of the requirement of strict logical precision does not, probably, take place in any science."

Last but not least are contradictions in mathematics. We all have reasons to think that the mystical fear of contradictions has been pushed to the background. The problem has lost its acuteness and probably its precision as well. The statements by Hao Wang (1961), a well-known expert in the axiomatic theory of sets, seem very interesting to me:

> 23. So far as the present state of mathematics is concerned, speculations on inconsistent systems are rather idle. No formal system which is widely used today is under very serious suspicion of inconsistency. The importance of set-theoretical contradictions has been greatly exaggerated. . . . But the more modern search for consistency proofs is differently motivated and has a more serious purpose than avoiding contradictions: it seeks for a better understanding of the concepts and methods.

A very interesting article by Hao Wang can hardly be rendered here, but it is noteworthy that it proves very lenient toward the problem of consistency. He remarks that contradictions are often very interesting, though they are never an object and nobody will recommend a method on the grounds that it is powerful enough to produce contradictions. At the same time, Hao Wang assumes the existence of a mathematical structure containing contradictions. If the latter are revealed in a system, that does not mean that the inferences logically obtained from the system are useless, since the corollaries may not make use of everything inherent in the initial structure. And if we are going to discuss practical problems, he goes on, e.g., building bridges, it is not at all necessary to formalize mathematics and prove its consistency, since there one can find many other more important and quite real problems. I believe that these statements affect essentially the views of those who are now dealing with the foundations of mathematics.

Now let us return to our everyday language. We have already said that it contains the elements of formal logic, and these are learned from childhood, while the language is being learned. At the same time, our everyday speech behavior is never completely logical. Try to concentrate upon the phrases of cursory dialogues, upon numerous advertisements and instructions; everywhere you will find illogicality. Moreover, if anybody tries to be absolutely logical in everyday speech behavior, he is immediately referred to the category of schizophrenics. Thus, the charm of Carroll's *Alice in Wonderland*, at least for adults, lies in the fact that the reader is introduced into the world of absurdly strict logic. Indeed, the word "smile" is a noun, and not an adjective, and, consequently, it is not illogical that a smile of the Cheshire cat can appear without the cat.

Another thing is interesting to note here. Psychiatrists are well aware of the fact that patients with a lowered intellectual activity cease to understand metaphors: they only perceive their literal meaning. A simple metaphor such as "to be in somebody's skin" receives a literal interpretation that is quite absurd from the standpoint of a normal person. One of the diagnostic symptoms in psychiatry is the failure of a patient to understand proverbs. On the other hand, another sign of lowered intellectual ability is a gross breach of logic; e.g., a patient is asked to interpret a series of deliberately nonsense pictures and he is unable to discover gross absurdity there. Or he is asked to arrange a series of connected pictures in a logical order, and he cannot do this; even if schizophrenics do fulfill this assignment, it is done in an oddly whimsical form. At the same time, patients begin to use words at will. They produce metaphors clear only to themselves (for details see Kasanin, 1944).

We may conclude that the intelligence of a normal person lies in a very narrow interval, limited on the one hand by failure to understand metaphors, fine alogical statements, and on the other hand by gross breaches of logic.

Semantic Scale of Languages

Classification of phenomena is one of the ways to describe a complex system. Many ways of classifying language systems can be suggested. I shall dwell here on only one of them: on constructing a semantic scale of languages. This system of classification will place languages in accordance with the role of the probabilistic structure of meaning.

Imagine a scale with one end occupied by *hard languages*, e.g., those of programming: there every sign possesses perfectly unambiguous, precise, and definite meaning—a mathematical or logical operation. This part of the scale will also be occupied by various dialects of the language of pure mathematics and mathematical logic, where symbols are used independently of the phenomena of the external world. The meaning of symbols is defined when they are introduced or it becomes clear after some statement, e.g., axioms, is formulated from them. In some cases, e.g., in mathematical logic and in the theory of the so-called context-free languages, no special sense related to the external world is ascribed to symbols.

On the other end of the scale, perfectly *soft languages* will be situated, in which the probabilistic structure of the meaningful content is manifested most explicitly. An example of such languages may be the language of abstract painting. In a separate study of abstract painting (Andrukovich et al., 1971), we have shown that its sign system may in fact be regarded as a language. Here, I shall say only that the sign system of this language has prior distribution functions which are deeply subjective. In this case, it is difficult to trace coordination which is usually observed for the prior distribution function of the meaningful content of everyday language. In this sense the language of abstract painting proves degenerate: the tendency observed in everyday language here becomes extreme.

Our everyday language and the languages of science are placed somewhere in the middle of the scale and occupy there a broad interval. In both these languages, prior functions of distribution of the word are to a certain extent coordinated, especially for people with similar intellectual orientation. But the degree of coordination varies over a wide range depending on the field of knowledge. This coordination is expressed to a minimal extent in the language of Western philosophy: hence emerged the conception of analytical philosophy with its statement that traditional philosophy is a pathology of language. Certain dialects of chemical language, above all, the language of chemical formulas, are perfectly unambiguous. If in a chemical text we come across the symbol "Na" it means only the metal sodium and nothing more, though in the formula NaCl the symbol Na takes on the meaning an ion of sodium. At the same time, in the language of science we face polymorphism which is sometimes expressed more strongly than in everyday language. This problem is discussed later (in Chapter 3), and here I shall only remark, getting slightly ahead, that the language of mathematics, when used to describe the phenomena of the external world, becomes polymorphous.

Quite a peculiar position on the semantic scale is occupied by the language of ancient Indian philosophy. There the right of words to be half-empty molds is acknowledged; everybody can fill them to his own taste, and contradictions are openly introduced into texts.

All this makes us place the language of ancient Indian philosophy on the semantic scale somewhere behind our everyday language, close to the language of abstract painting. Later, I shall return to analyzing this language. Here I should like to draw attention to the point that from my position the classification of languages, in the case of a broad formulation of the problem, should be made not according to the people speaking them but according to epochs of culture: languages of two peoples (e.g., English and French) belonging at present to one, general European culture will not occupy different places on the semantic scale.

The semantic scale may also be presented as an open straight line, assuming that soft languages are tending to its one side and hard ones to the other. Then, according to topological considerations, it will follow that an addition of a distant point will allow us to turn a line into a circle. Such a distant point is the language of the religious–philosophical system Zen, a Japanese branch of Buddhism which has very little in common with the original teaching. The language of Zen is a strange language of absurd statements. The latter are built as illogical phrases, "koan," containing only hints. These are riddles without rational solutions. An adept of a Zen monastery would have to be immersed in meditation in order to reveal the "sense" strangely coded in unusual sentences. Months or even years may be spent on this. Here are several examples of such statements borrowed from the thesis of Pomerants (1968).

> Does a dog have the Buddha nature? Nothing!
>
> Two hands make a clap; and what is the sound of one hand?
>
> When much is reduced to one, to what can one be reduced?
>
> Call it a stick, and you state it; don't call it a stick, and you deny it.
> So, not stating and not denying now will you call it? Speak! Speak!

These statements cannot be called logically contradictory. If we use Wittgenstein's terminology, they can probably be called senseless, or, even better, prohibited from the standpoint of our everyday language. But, as a matter of fact, this is a language conveying certain strange profound sense. When a koan is solved, it becomes clear for an adept that it is a simple, clear and almost self-evident statement made by the teacher in the state of ecstasy. The teacher wants his pupil to achieve a similar lucid state, and for this reason he resorts to constructing statements.

Of course, Zen is not only a set of koan; it is something more—an outlook which has made a strong impresson upon the whole of Japanese culture (for details see Grigorieva, 1971). The influence of Zen on Western culture is easily traced in the works of such painters as Van Gogh and Henry Matisse, the writer J. D. Salinger, and, in its vulgar form, American beatniks, e.g., in the characters of Jack D. Kerouac. All this is well presented in the book by Zavadskaya (1970). But here we are interested in something else, namely, typological community of phenomena in Western and Eastern cultures which have emerged independently and without contact, according to the terminology of Konrad (1966). I believe that in the language of Western culture we can observe the tendencies which are manifested most vividly in the language of Zen. First of all, there are the metaphors of our language: they introduce into our speech the same shocking flavor of incompatibility which in its refined form is manifested in koan. In addition, I draw the reader's attention to some proverbs, sayings, captions to cartoons, and anecdotes, especially abstract ones.[8] In a grotesque form it is manifested in surrealistic painting. I am looking now at a reproduction of the picture "Invention of Monsters" by Salvador Dali from the Chicago museum. To the right there is a giraffe on fire; in the center, a table on which there is a sculpture of a horse head with female breasts; at the table, an almost human figure with little wings; in the right upper corner, nude figures in ridiculous postures; in the left lower corner, an odd group of people with an air of conspiracy; and in the right lower corner, a puppy. Each of the compositional constituents is painted quite realistically: both the giraffe and the fire look very realistic. The whole composition, however, is a riddle like a koan. It is its incompatibility that shocks the spectator.

Thus, our semantic scale is closed at the point where the language of Zen, the most unusual human language of all, is situated. And what seems especially interesting, in this language, is that the tendencies which are most explicit manifest themselves to a certain degree, sometimes very vividly, in the expressive means of other cultures.

Koan, in their logical structure, resemble very much the antiplays by E. Ionesco and S. Beckett (the reader can acquire a certain notion of them from Kulikova, 1970). Here, as well as in abstract painting, "the reverse side of logic" is used.

General Philosophical Prerequisites of the Probabilistic Model of Language

Our probabilistic model of language does not need strict philosophical prerequisites. When one is considering the question of language, it is not necessary to proceed from a very strong statement of world cognizability.[9] Without considering this statement in depth and without denying it, I shall only point to the fact that we shall be satisfied with the statement that our language should be capable of describing our idea of the world, which constantly develops and grows.

There is no need either to state with certainty that the mechanism of thinking is arranged in some particular way. Traditionally, it seems plausible to assume the existence of a hierarchy of thinking: (i) pre-logical, imaginative thinking [largely restricted to primitive civilizations (Lévi-

[8] Here is an abstract anecdote:
"I say, there are bananas in your ears."
"Sorry, I can't hear you: I have bananas in my ears."
It is not clear where these anecdotes have come from: whether they have come from the West or appeared independently.

[9] I consider this question more thoroughly in my book *Faces of Science*, published in Russian in 1976.

Strauss, 1964)]; (ii) logical thinking; (iii) superlogical thinking whose mechanism remains unknown. Obviously, people may belong to various levels of the thinking hierarchies. However, communication, especially scientific communication, is preferably carried out on the logical level. Deductive logic is to a great extent a means of communication rather than a means of thinking. The task of logic is a development of ideas which are contained in a condensed, and for this reason not quite clear, form in the original premises. This is especially well seen in the language of mathematics where the deductive structure of constructing judgments is traced most easily. Here, I should like to quote the words of de Broglie (1960), the well-known French physicist:

> "The language of mathematics, due to its rigidly deductive character, allows us to give a detailed description of intellectual values already obtained, but it does not allow us to obtain novel values, So, it is not pure deductions but bold inductions and original concepts which are the source of the great progress in Science."

While logic is a means of communication, language polymorphism is the way to overcome difficulties in a logically built system of communications rather than in the system of thinking. (We must divide thinking proper and the means of expressing it.) The probabilistic model of language is merely one possible explanation of the way the difficulty may be overcome.

At the same time, the fuzzy nature of our language makes us ponder the structure of our consciousness. But this is quite a separate subject which I shall briefly examine in Chapter 8.

I should like to draw the reader's attention to a certain parallel in the development of physics and linguistics. The concept of atomic word meanings, which seems to go back as far as Leibniz (or probably even the Cabala), was given substantial support by Frege, Russell, and the early work of Wittgenstein simultaneously with the seemingly finite conclusion of the atomic nature of matter clearly localized in space and time. At present, as a result of the progress of quantum mechanics, we are dealing with a fuzzy nature of subatomic particles. Here is how the physicist Capra (1976) in his paper devoted to comparing the ideas of modern physics with the ancient Oriental outlook attempts to sum it up:

> One of the main insights of quantum theory has been the recognition that probability is a fundamental feature of the atomic reality which governs all processes, and even the existence of matter. Subatomic particles do not exist with certainty at definite places, but rather show—as Heisenberg (1963) has put it—"tendencies to exist." Atomic events do not occur with certainty at definite times and in definite ways, but rather show "tendencies to occur." Henry Stapp

(1971) has emphasized that these tendencies, or probabilities, are not probabilities of "things," but rather probabilities of interconnections.

> Any observed atomic "object" constitutes an intermediate system connecting the preparation of the experiment and the subsequent measurement. The properties of the object cannot be defined independently of these processes. If the preparation or the measurement is modified, the properties of the object will also change. (p. 22)

In the probabilistic model of language, the fundamental thing is a probabilistical setting of the text meaning. The prior distribution function of the word meaning $p(\mu)$ is but a "tendency for the word meaning to be realized," is a preparation to an experiment carried out in verbal behavior by constructing a concrete phrase. The likelihood function $p(y|\mu)$ arising while reading the phrase is a direct analogue of a physical measurement, as I have already mentioned in my discussion of the Bayesian model of language. The meaning of the text arises as a probabilistic description of the interacting "readiness to comprehend"—a "verbal experiment" aimed at comprehension.

The analogy proves to be a profound one. It is probably pertinent to say that the probabilistic model of language has resulted from a paradigm of modern physics. It turns out that both the concept of discrete subatomic particles in physics and that of discrete words of our language are but a conventional denotation of what is developed within the context which is at one time given by a physical experiment and at another time by a common phrase of an everyday dialogue.

We might continue the analogy by comparing words with hadrons—strongly interacting particles generating almost all of the subatomic particles known at present. Here is another quotation from Capra (1976):

> The important new concept in S-matrix theory is the shift of emphasis from objects to events. Its basic concern is not with the particles, but with their reactions. Such a shift from objects to events is required both by quantum theory and by relativity theory. On the one hand, quantum theory has made it clear that a subatomic particle can only be understood as a manifestation of the interaction between various processes of measurement. It is not an isolated object, but rather an occurrence, or event, which interconnects with other events in a particular way. Relativity theory, on the other hand, has forced us to conceive of particles in terms of space-time, as four-dimensional patterns, processes rather than objects.
>
> The S-matrix approach combines both of these viewpoints. Using the four-dimensional mathematical formalism of relativity theory, it describes all properties of hadrons in terms of reaction probabilities, and thus establishes an intimate link between particles and processes.

> Each reaction involves particles which link it to other reactions and thus build up a whole network of processes. (p. 28)
>
> The picture of hadrons which emerges from these bootstrap models is often summed up in the provocative phrase: "Every particle consists of all other particles." It must not be imagined, however, that each hadron contains all the others in a classical, static sense. Rather than "containing" one another, hadrons "involve" one another in the dynamic and probabilistic sense of S-matrix theory, each hadron being a potential bound state of all sets of particles which may interact with one another to form the hadron under consideration. In that sense, all hadrons are composite structures whose components are again hadrons, and none of them is any more elementary than the others. The binding forces holding the structures together manifest themselves through the exchange of particles, and these exchanged particles are again hadrons. Each hadron, therefore, plays three roles: it is a composite structure, it may be a constituent of another hadron, and it may be exchanged between constitutents and thus constitute part of the forces holding a structure together. (p. 36)

For language the analogy is striking. Words in dictionaries are explained by other words, but this is not to say that the meaning of a word consists of the meanings of the words by which it is explained. Phrases are composed with words, probabilistically interacting with one another. This is a phrase structure generating a new sense which is missing from the constituent words though, somehow, they "contain" it.

Concluding Remarks

In concluding this chapter, I should like to say the following. Humanity seems to have always realized the *insufficiency of its means of communication.* Human thinking, and more broadly human inner life, is evidently richer than language is. This idea has been expressed in various ways by many people. For example, a line from the poem "Silentium" by Tyutchev says, "A thought once uttered is untrue." In the poem "**Благославляю все, что было**" ("I Bless All That Happened") by A. Blok, we find the lines:

> **Все, чего не скажешь словом,**
> **Узнал я в облике твоем**
> (All that can't be said in words
> I saw in your image)

In Schopenhauer (1862) we read:

> A thought lives while it is verbalized: then it is petrified and hence-

> forth remains dead but unperishing like petrified primeval animals and plants. Its momentary life may also be compared to a crystal at the instant of its creation.
>
> And as soon as our thought has clothed itself into words, it loses its heartiness and profound significance. Starting to exist for others, it stops living within ourselves, – as a child who, separating himself from his mother, enters his own existence. (p. 74–75)

In the words of John Ruskin:

> To explain is to waste time. A clear-sighted man catches your hint; but an ill-sighted man will not comprehend after a long speech. (cited from Tolstoy, 1905, p. 191)

Heidegger stated:

> A "true" man speaks "truly" only when he keeps silent. (cited from Stassen, 1973, p. 43)

Jung (1930) expressed the same idea:

> One of the greatest mistakes of our culture . . . is an intense belief in words and exposition and infinite over-estimation of teaching by words and methods. (p. 88, my translation)

Zavadskaya (1970) cited the words of Matisse's teacher: "Regard painting as passionate silence." Remember the "noble silence of Buddha" with which he answered difficult questions. Well-known is the concept of word insufficiency in the "theory of silence" of Chuang-Tzu, one of the founders of Taoism. He said: "The sound of an unspoken word is louder than the thunder of a drum" (*Drevnekitaiskaya Filosofiya*, 1972, my translation). An important role is ascribed to silence in the religious philosophy of Yoga (Swami Sivananda, 1967): "Listen to soft, hardly audible voices of silence. . . . The power of silence infinitely exceeds the power of lectures, talks, speeches and discussions. . . . The language of silence is the language of God . . ." (my translation).

Hillel, a character in the novel *Golem* by Meyrink, says, "Do you think our Jewish books are written only in consonants just accidentally? Everyone has the opportunity to insert those vowels which will help him to reveal the mysterious sense intended only for him alone—otherwise the live word would have turned into a dead dogma" (my translation).

Doubts about word meanings have manifested themselves most vividly in the teaching of Zen. In modern times, this idea has been clearly formulated by the Indian thinker Krishnamurti.

> Understanding does not come with knowledge. In the interval between words, between thoughts, comes Understanding, – this interval is silence unbroken by knowledge, it is the open, the unponderable, the implicit. (Pomerants, 1965)

In Wittgenstein's paper we read: "Whereof one cannot speak, thereof one must be silent"; this is the concluding paradox of the *Tractatus*.

However, people are always searching for a new language. New languages mean new cultures. And I believe that the statement that the history of human culture is the history of sign systems is perfectly correct.

Scientific development is also reflected in the development of a scientific language. According to Hutten (1956), "Science is a linguistic or symbolic representation of experience."

Almost the same formulation can be found in the book by Kopnin (1971): "Language is a form of knowledge of existence as a system of signs."

In Langer's (1951) book we read: ". . . the edifice of human knowledge stands before us, not as a vast collection of sense reports, but as a structure of *facts that are symbols and laws that are their meanings.*"

In the book *Marxism and the Philosophy of Language* by Voloshinov (1929), an even stronger statement is made: "Everything ideological possesses meaning: it presents, describes, replaces something outside it, i.e. is a sign. Where there is no sign there is no ideology."

Physicists also have asserted the insufficiency of modern linguistic means. For example, Heisenberg (1958), describing the impetuous reaction to modern physics development, says:

> . . . it probably means that one has not yet found the correct language with which to speak about the new situation and that the incorrect statements published here and there in the enthusiasm about the new discoveries have caused all kinds of misunderstanding. This is indeed a fundamental problem. The improved experimental technique of our time brings into the scope of science new aspects of nature which cannot be described in terms of the common concepts. But in what language, then, should they be described? . . . However, if one wishes to speak about the atomic particles themselves one must either use the mathematical scheme as the only supplement to natural language or one must combine it with a language that makes use of a modified logic or of no well-defined logic at all.

Peculiar features of a culture are most expressively reflected in the "language" of its architecture. Buildings are phrases of this language built from separate constructive elements–signs forming an alphabet of the language. Ensembles of building are texts of this language. The hierarchical structure of the language of architecture is quite obvious.

People are permanently searching for new forms of expression and sometimes find such sudden means as Zen. Language, having received an impetus to its existence, begins to develop as a self-organized system influencing human thinking. It seems rash to decide what is here of primary or secondary importance: it is only language that we can observe and analyze phenomenonologically. For this reason, it is more convenient to speak of one system: of the language of culture, and of its sign system. Here I should like again to cite Wittgenstein (1955): "The *limits of my language* mean the limits of my world" (paradox 5.6).

However, the same thought has been formulated much earlier by Humboldt (1843), who stated that the difference between languages is the difference between outlooks. The statement of Whorf (1956) that we perceive nature the way it is expressed in our native language sounds very similar. If language is viewed as an instrument, then, speaking of its impact upon the formation of our concepts, we may make an analogy with the impact of an instrument of measurement on the results of measurements in the microworld. (This comparison was suggested by S. K. Shaumyan.)

Even if we adhere to the point of Leibniz and believe in the existence of necessary logical truths that remain true in all possible Worlds, still they are not charged with any information about our World and the way we perceive it. The question remains open whether, and to what degree, we can understand languages of other cultures, say, the language of ancient Indian philosophy or the language of Zen. Many people now feel the necessity to enrich European culture with new ideas, to respiritualize it—hence the interest in other cultures and the languages by means of which they are expressed.

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