Highly Cited Works in Mathematics. Part 2.
"Applied" Mathematics.

In a previous editorial ${ }^{1}$ I presented a list of highly cited works in "pure" mathematics-books and journal articles cited in the Science Citation Index ${ }^{*}$ ( $S C I^{\circ}$ ) during the period 1961-1972. At the end of this editorial, you will find a similar list of books and journal articles in "applied" mathematics, heavily cited in the same 1961-1972 period. I refer the reader to the previous editorial for a detailed explanation of how these lists were compiled from ISI *'s data base of 30 million citations.

Briefly, we first identified highly cited books and journal articles on mathematics and its applications in research, and then classified the items retrieved as either "pure" or "applied" mathematics. Classification was accomplished by a computer algorithm. ${ }^{2}$ To make it as "pure" mathematics, the algorithm required that an item have been cited in 1972 at least 4 times by articles from mathematics and statistics journals covered by the $S C I$. It required further that the item be cited in common with some other item by at least 3 articles in those same 1972 issues. Articles that did make it were classified as "pure" mathematics, and were listed in the editorial noted above. Others were classified as "applied" mathematics and they are listed on pages 7.9 following.

There will undoubtedly be differences of opinion about whether a particular item belongs on one list or the other, or on either list at all. But the heavy citation is undoubtedly significant as a whole-though undoubtedly for different reasons in connection with different items-and we offer this
list, as we did that on "pure" mathematics, as another example of statistical bibliography (bibliometrics).

There are 72 items on the list (space in these pages limited the list to articles cited 90 or more times during the 1961-1972 period). Of the 72 items, 46 (64\%) are books, and $26(36 \%)$ are journal articles. ${ }^{3}$ A rough analysis of my own shows that of the 72 items, about $39 \%$ deal with statistics generally, about $27 \%$ with biometrics specifically, about $17 \%$ with mathematical physics and technometrics, and about $17 \%$ with what I take to be analyses of specific mathematical topics for varied application. ${ }^{4}$

The ranking of these items will not, I hope, become a matter for "subjectiveanalytical" contention. The ranking is by citation totals over a twelve-year period. Although I believe the citation totals indicate much, I know-as I have said and written many times before and shall continue to say and write-that they do not indicate everything. I hope all will agree, however, that they suggest worthwhile subjects for speculation and study. Why does one item rank above another?

For example, Snedecor's well-known manual ${ }^{5}$ leads the list. The book is responsible for the appearance of Professor Snedecor's name on other "mostcited" lists we have compiled. 6 I am sure it will neither surprise nor offend Professor Snedecor if I say that there may be other items lower ranked on the list that probably exceed his book in scholarly merit in one way or another, if one is thinking in terms of the development of statistical theory or
some such thing. His widely-used handbook was written for the benefit of research workers and students of statistics. It was written to make statistics accessible to non-mathematicians and non-statisticians for improvement of their experimental methodology. The citation record shows (as does the book's publication history) that it has been used as it was written to be used, by those who needed an explanation of statistical method, and examples of its application in their own fields. In other words, the citation record records the impact the book has indeed had, the impact 1 am sure its author hoped it would have, in improving the rigor of experimental analysis. As such it can surely be said to have made a "contribution" to statistics that perhaps more scholarly works have not-and perhaps won't until some other Snedecor digests them and makes them accessible for application by non-statisticians. It is this kind of speculation and examination that I hope these lists will inspire.

1. Garficld, E. Highly cited works in mathematics. Part 1. "Pure" mathematics. Current Contents ${ }^{\text {® }}$ No. 47, 21 November 1973, p. 5.9.
2. I am glad to acknowledge again the aid of Professor Kenneth O. May of the University of Toronto for his comments on problems involved in differentiating between the "pure" and "applicd:"
3. An interesting comparison is that of 78 items on the "pure" mathematics list, $56(72 \%)$ were books and only $22(28 \%)$ were journal articles.
4.     - In cases of the last, especially, I suspect some readers may think that an item should have appeared on the list of "pure" mathematics. We have listed them here, however, as "instructed" by our rigorous algorithm.
5. G.W. Snedecor \& W.G. Cochran, Statistical methods. 6th ed. Ames: Jowa St. Univ. Press, 1967.
6. For example, see: Garficld, E. More on forecasting Nobel Prizes and the most cited scientists of 1972. Current Contents No. 40, 3 October 1973, p. 5-7.-- In 1967 Sncdecor ranked as 10 th most cited author ( 904 citations) ; in 1972 as 5th (1185 citations) ; and for the period 1961-1972 as 7th ( 10,279 citations). See also: Garficld, E. Play the new game of twenty citations: wherein ISI reveals the fifty most frequently cited "non-journal" items. Current Contents

One thing is certain, however. As a whole the list shows the critical role played by statistics in not only the "hard" sciences, but in the social sciences as well. 7 It is probably impossibl to overemphasize the role of statistics for the future of research in the social sciences and for the future development of social policy. Sociologists continually complain of the lack of data, or of the lack of proper statistical analysis of available data, in historical and sociological studies. ${ }^{8}$ In fact, it may be (as some would complain) difficult to think of sociological research within any other framework but the statistical. Earlier this year, W. Kruskal reminded us of "that huge cloud of statistical thought and action that suffuses all government activity ... " and called for an improvement of communication between statisticians working in various areas of the social sciences. 9 Perhaps citation analysis and lists like that which follows, may help in this respect as well. $1^{10}$
No. 32, 11 August 1971, p. 5.9.-Among "non-journal" items cited in 1967, Snedecor's Statistical Methods (with 880 citations) was the most frequently cited monograph.
7. This dual role of statistics presented something of a dilemma in establishing criteria for inclusion of journals in ISI's Social Sciences Citation Index ${ }^{\mathrm{TM}}$ (SSCI ${ }^{\mathrm{TM}}$ ). To have included all statistical journals would have put into the SSCI thousands of irrele vant articles on statistical methodology in the hard sciences. Our solution was to select articles from non-biometrics and non-social science journals if they cited two or more articles from social-science journals.
8. See, for example, almost any chapter of S.M. Lipset, Revolution and Counter Revolution: Change and Persistence in Social Structures. New York: Basic Books, 1968.
9. Kruskal, W. The Committee on Na tional Statistics. Science 180(4092): 1256.8, 1973.
10. As regards the importance of statistics for the social sciences, it is interesting to note the membership in the Allied Social Sciences Associations (ASSA) of the American Statistical Association, the Biometrics Socicty, and the Institute of Mathematical Statistics as well. The ASSA is holding its 1973 Annual meeting in New York, December 27-30.

# A List of Highly Cited Works in Mathematics, 1961-1972. Part 2. "Applied" Mathematics 

| TotalCitations |  |  |  |
| :---: | :---: | :---: | :---: |
| $\underset{\underset{\sim}{E}}{\underline{E}}$ (1) |  | $\underset{\sim}{\mathbf{N}}$ <br> (3) | Bibliographical Data (4) |
| 1. | 3254 | 795 | Snedecor, G.W. \& Cochran, W.G. Statistical methods. 6th ed. Iowa, 1967. |
| 2. | 2770 | 273 | Duncan, D.B. Multiple range and multiple I: tests. Biometrics. 11: 1-42, 1955. |
| 3. | 1824 | 470 | Siegel, S. Nonparametric statistics for the behavioral sciences. McGraw, 1960. |
| 4. | 1514 | 524 | Steel, R.G. \& Torrie, J.H. Principles and procedures of statistics. McGraw, 1960. |
| 5. | 1444 | 471 | Winer, B.J. Statistical principles in experimental design. McGraw, 1962. |
| 6. | 622 | 199 | Abramowitz, M. \& Stegun, I.A., eds. Handbook of mathematical functions with formulas, graphs and mathematical tables. Dover, 1964. |
| 7. | 432 | 79 | Cooley, J.W. \& Tukey, J.W. An algorithm for the machine calculation of complex Fourier series. Math. Comput. 19(90):297-301, 1965. |
| 8. | 398 | 52 | Kramer, C.Y. Extension of multiple range tests to group means with unequal nuinbers of replications. Biometrics. 12:307.310, 1956. |
| 9. | 341 | 102 | Dixon, W.J. \& Massey, F.J.Jr. Introduction to statistical analysis. 3rd ed. McGraw, 1969. |
| 10. | 322 | 23 | Wigner, E.P. On unity representations of the inhomogenous Lorentz group. Ann. Math. 40:149-204, 1939. |
| 11. | 315 | 45 | Lindquist, E.F. Design and analysis of experiments in psychology and education. HM, 1956. |
| 12. | 307 | 75 | Finney, D.J. Statistical methods in biological assay. Hafner, 1964. |
| 13. | 299 | 90 | Crank, J. Mathematics of diffusion. Oxford, 1956. |
| 14. | 264 | 50 | Cochran, W.G. \& Cox, G.M. Experimental designs. Wiley, 1957. |
| 15. | 251 | 30 | Cochran, W.G. Some methods for strengthening the common chisquare tests. Biometrics. 10:417-451, 1954. |
| 16. | 247 | 55 | Edwards, A.L. Lxperimental design in psychological research. HR\&W, 1968. |
| 17. | 247 | 38 | Dunnett, C.W. A multiple comparison procedure for comparing several treatments with a control. J. Amer. Statist. Assoc. 50:1096-1121, 1955. |
| 18. | 246 | 53 | Finney, D.J. Probit analysis. Cambridge, 1971. |
| 19. | 237 | 85 | Sokal, R.R. \& Sneath, P.H. Principles of numicrical taxonomy. Freeman, 1963. |
| 20. | 237 | 12 | Hays, W.L. Statistics for psychologists. HR\&W. 1963. |
| 21. | 234 | 55 | Watson, G.N. Theory of Bessel functions. Cambridge, 1944. |
| 22. | 232 | 31 | Fisher, M.E. Correlation function and the critical region of simple fluids. J. Math. Phys. 5:944-962, 1964. |
| 23. | 225 | 28 | Fisher, R.A. Statistical methods for rescarch workers. Hafner, 1969. |
| 24. | 211 | 92 | Draper, N. \& Smith, H. Applied regression analysis. Wiley, 1966. |

A List of Highly Cited Works in Mathematics, 1961-1972. This list shows, in order of decreasing frequency of citation, works in applied mathematics highly cited during the period 1961-1972. Column 1 shows the rank of the item on this list. Column 2 shows the total number of times the item was cited by journals indexed in the Science Citation Index during the period 1961-1972. Column 3 shows the number of times the item was cited in 1972 only. Column 4 gives full bibliographic data. Publisher information in the case of books has been abbreviated. Full information is as follows (as a matter of interest we have included in

|  |  | N <br> $\mathbf{N}$ <br> (3) | Bibliographical Data (4) |
| :---: | :---: | :---: | :---: |
| 25. | 211 | 12 | Mann, H.B. On a test of whether one of two random variables is stochastically larger than the other. Ann. Math. Statist. 18:50-60, 1947. |
| 26. | 210 | 16 | Newton, R.G. Analytic properties of radial wave functions. J. Nath. Phys. 1:319.347, 1960. |
| 27. | 206 | 10 | Lehmann, H. Uber Eigenschaften von Ausbreitungsfunktionen und Renormierungskonstanten quantisierter Felder. Nuovo Cimento. 11:342-357, 1954. |
| 28. | 194 | 14 | Suzuki, M. Consequences of current commutation relations in the nonleptonic hyperon decays. Thys. Rev. Letters. 15:986.989, 1965. |
| 29. | 189 | 38 | Fisher, R.A. Dispersion on a sphere. P. Roy. Soc. Lundon A. 217:295. 305, 1953. |
| 30. | 185 | 42 | Ran. C.R. Advanced statistical methods in biometric research. Hafner, 1952. |
| 31. | 184 | 24 | Penrose, R. A generalized inverse for matrices. Camb. Phil. Soc. Proc. 51:406-413,1955. |
| 32. | 183 | 12 | Cutkosky, R.E. Singularities and discontinuities of Feynman amplitudes. J. Math. Phys. 1:429-433, 1960. |
| 33. | 162 | 23 | Fisher, R.A. \& Yates, F. Statistical tables for biological, agricultural and medical research. Hafner, 1964. |
| 34. | 160 | 87 | Sokal, R.R. \& Rohlf, F.J. Biometry, the principles and practice of statistics in biological research. Freeman, 1969. |
| 35. | 160 | 0 | Bargmann, V. \& Wigner, E.P. Group theoretical discussion of relativistic equations. P. Natl. Acad. Sci. USA. 34:211.223, 1948. |
| 36. | 143 | 32 | Hooke, R. \& Jeeves, T.A. "Direct search" solution of numerical and statistical problems. J. Assoc. Comp. Mach, 8:212-229, 1961. |
| 37. | 139 | 50 | Papoulis, A. Probability, random variables and stochastic processes. McGraw, 1965. |
| 38. | 138 | 17 | Wald, A. Sequential analysis. Wiley, 1947. |
| 39. | 131 | 12 | Peaceman, D.W. The numerical solution of parabolic and elliptic differential cquations. J. Soc. Ind. Appl. Math. 3:28-41, 1955. |
| 40. | 125 | 26 | Lehman, E.L. Testing statistical hypotheses. Wiley, 1959. |
| 41. | 122 | 33 | Eden, L.J. et ul. Analytic S-matrix. Cambridge, 1966. |
| 42. | 122 | 20 | Middleton, D. Introduction to statistical communication theory. McGraw, 1960. |
| 43. | 122 | 20 | Walker, H.M. \& Lev, J. Statistical inference. HR\&W, 1953. |
| 44 | 122 | 15 | Slepian, D. Prolate spheroidal wave functions. Fouricr analysis and uncertainty. Bell Syst. Tech. J. 40:43-46, 1961. |
| 45. | 122 | 14 | Grad, H. On the kinetic theory of rarefied gases. Comm. Pure Appl. Math. 2:331-407, 1949. |
| 46. | 121 | 31 | Davis, P.J. Interpolation and approximation. Blaisdell, 1963. |
| 47. | 120 | 38 | Hill, R. Mathematical theory of plasticity. Oxford, 1950. |

parentheses after the listed abbreviation the number of items each publisher contributed to the list):
AW (1) Reading, Mass.: Addison-Wesley Publishing Co., Inc. Blaisdell (1) New York:
Blaisdell Publishing Co., Inc. Cambridge (4) New York: Cambridge University Press Chicago
(1) Chicago: University of Chicago Press Dover (3) New York: Dover Publications, Inc.

Freeman (2) San Francisco: W.H. Freeman \& Co. Hafner (4) New York: Hafner Publishing

| TotalCitationa |  |  |  |
| :---: | :---: | :---: | :---: |
| 曾 |  | $\stackrel{\text { N }}{\mathbf{\sigma}}$ | Bibliographical Data |
| (1) | (2) | (3) | (4) |
| 48. | 119 | 21 | Hartley, H.O. The modified Gauss-Newton method for the fitting of non-lincar regression functions by least squares. Technometrics. 3:269 280, 1961. |
| 49. | 116 | 39 | Nalimov, V.V. Application of mathematical statistics to chemical analysis. AW, 1962. |
| 50. | 114 | 32 | Richtmyer, R.D. Difference me thods for initial value problems. Wilcy, 1967. |
| 51. | 114 | 21 | Riesz, F. \& Nagy, B.S. Functional analysis. Ungar, 1955. |
| 52. | 114 | 0 | Sharinon, C.E.Communication in the presence of noise. PIRE. 37:10-21, 1949. |
| 53 | 113 | 33 | Hardy, G.H. et al. Inequalities. Cambridge, 1952. |
| 54. | 111 | 40 | Hildebrand, F.B. Introduction to numerical analysis, McGraw, 1956. |
| 55. | 108 | 31 | Sneddon, I.N. Fourier transformations. McGraw, 1951. |
| 56. | 108 | 15 | Kac, M., Uhlenbeck, G.E. \& Hemmer, P.C. On the Van der Waals theory of the vapor-liquid equilibrium. 1. Discussion of a one-dimensional model. J. Math. Phys. 4:216-223, 1963. |
| 57. | 105 | 26 | Shannon, C.E. \& Weaver, W. Mathematical theory of communication. Illinois, 1949. |
| 58. | 104 | 29 | Cooley, W.W. \& Lohnes, P.R. Multivariate data analysis. Wiley, 1971. |
| 59. | 101 | 34 | Harmon, H.H. Modern factor analysis. Chicago, 1967. |
| 60. | 101 | 25 | Titchmarsh, E.C. Theory of functions. Oxford, 1939. |
| 61. | 101 | 10 | Henderson, C.R. Estimation of variance and covariance components. Biometrics. 9:226-252, 1953. |
| 62. | 98 | 38 | Bailey, N.T.J. Statistical methods in biology. Wiley, 1959. |
| 63. | 98 | 32 | Erdelyi, A. Asymptotic expansions. Dover, 1961. |
| 64. | 95 | 15 | Duncan, D.B. Multiple range tests for correlated and heteroscedastic means. Biometrics. 13:164-176, 1957. |
| 65. | 94 | 32 | Goldstein, A. Biostatistics. Macmillan, 1964. |
| 66. | 94 | 14 | Box, G.E.P. \& Wilson, K.B. On the experimental attainment of optimum conditions. J. Roy. Stat, Soc. B. 13:1-45, 1951. |
| 67. | 93 | 35 | Harrington, R.F. Field computation by moment me rhods. Macmillan, 1968. |
| 68. | 93 | 17 | Powell, M.J.D. Method for minimizing a sum of squares of non-lincar functions without calculating derivatives. Computer J. 7:303-307, 1965. |
| 69. | 91 | 42 | Morrison, D.F. Multivariate statistical methods. McGraw, 1967. |
| 70. | 91 | 25 | Bendat, J.S. \& Peirson, A.G. Measurement and analysis of random data. Wiley, 1966. |
| 71. | 91 | 18 | Jahnke, E. \& Emde, F. Tables of functions with formulae and curves. Dover, 1945. |
| 72. | 90 | 23 | Bellman, R. Introduction to matrix analysis. McGraw, 1960. |

Co., Inc. HM (1) Boston: Houghton Mifflin Co. HR\&W (3) New York: Holt, Rinehart \& Winston, Inc. Illinois (1) Urbana: University of Illinois Press Lowa (1) Ames: Iowa State University Press Macmillan (2) New York: Macmillan Company McGraw (10) New York: McGraw-Hill Book Company Oxford (3) New York: Oxford University Press Ungar (1) New York: Frederick, Ungar, Publishing Co., Inc. Wiley (8) New York: John Wiley \& Sons, Inc.

