CC/NUMBER 44
NOVEMBER 2, 1992

# This Week's Citation Classic®

**van den Bos A.** Alternative interpretation of maximum entropy spectral analysis. *IEEE Trans. Inform. Theory* 17:493-4, 1971.
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Maximum entropy spectral analysis is a method for the computation of the power spectrum of a stochastic process from a number of autocovariance lag values. From all spectra exactly compatible with the lag values, the method selects the one that maximizes the entropy of the process. It is shown that the maximum entropy spectrum is equivalent to the conventional autoregressive spectrum computed from the same lag values. [The *SCI*® indicates that this paper has been cited in more than 135 publications.]

## For the Fun of It: A Simple Explanation for Maximum Entropy Spectral Analysis

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In August 1968, I attended the NATO Advanced Study Institute on Signal Processing with Emphasis on Underwater Acoustics, held at Twente Institute of Technology, Enschede, The Netherlands. At this conference, J.P. Burg presented his now-famous paper "A new analysis technique for time series data"[1] about maximum entropy spectral analysis. The message of Burg's paper was that the power spectrum of a stochastic process could be computed with infinite resolution from a finite number of autocovariance lag values. This greatly puzzled the audience that had been brought up with classical nonparametric spectral analysis[2] and had been taught that infinite resolution requires an infinite number of lag values. After the session, the general feeling was accurately expressed by my friend Huibert Kwaakernak saying to me that infinite resolution from a finite number of lag values reminded him of what he called the Von Münchhausen trick. He was referring to the German nobleman who, in his memoirs, tells how he lifted himself and the horse he was sitting on from the swamp by his own hair.

After the conference the maximum entropy method kept intriguing me and, although I am not a specialist in spectral analysis, I tried to find a simple explanation for it. The one I found, and which is proposed in the paper, may be described as follows. From all spectra exactly compatible with the available lag values, Burg's method selects the one that maximizes the entropy of the process. In my paper, maximum entropy spectral analysis is alternatively interpreted, in the time domain, as stepwise extrapolation of the available lag values so as to maximize the entropy in each step. A modest amount of calculus then suffices to show that the resulting extrapolated autocovariance and the corresponding spectrum are identical with those of a conventional autoregressive process exactly compatible with the available autocovariance lag values. This parametric, autoregressive interpretation does not require any entropy considerations.

The absence of entropy considerations may well be a reason why the paper is so frequently cited, but I think it is not the only one. The fact that the paper, most unusually, consists of only slightly more than one printed page and that use is made of only elementary mathematics has undoubtedly also contributed to its popularity. I think the material of my paper has nowadays been incorporated in the literature.[3,4] Its role in the development of spectral analysis has been described in detail by D.G. Childers in the introduction to the collection of reprints of papers on spectral analysis edited by him.[5] A reprint of my paper is also included in this collection.

The effects of the paper for me personally were all positive. In particular, it resulted in interesting contacts and lasting cooperation with colleagues. But above all, it taught me now and then to leave the main line of my research for a while. Just for the fun of it.

1. **Burg J P.** A new analysis technique for time series data. Paper presented at the NATO Advanced Study Institute on Signal Processing with Emphasis on Underwater Acoustics, Enschede, The Netherlands, 1968. (Reprinted in: Childers D G, ed. *Modern spectral analysis.* New York: IEEE Press, 1978. p. 42-8.) (Cited 195 times.)
2. **Blackman R B & Tukey J W.** *The measurement of power spectra from the point of view of communication engineering.* New York: Dover, 1959. 190 p. (Cited 700 times.)
3. **Therrien C W.** *Discrete random signals and statistical signal processing.* Englewood Cliffs, NJ: Prentice-Hall, 1992. 727 p.
4. **Papoulis A.** *Probability, random variables, and stochastic processes.* New York: McGraw-Hill, 1991. 666 p.
5. **Childers D G.** Introduction. (Childers D G, ed.) *Modern spectral analysis.* New York: IEEE Press, 1978. p. 1-4.

Received August 10, 1992