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This Week's Citation Classic

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The paper has had a great effect in several directions. First of all, it has led to software for solving Poisson's equations over rectangular domains, and for equations where the method of separation of variables is applicable. In addition, it' s been a basic tool for solving problems associated with the technique known as domain decomposition. Here one takes the union of rectangular domains and over each subdomain one solves a problem exactly. Having done so, it is necessary to "paste" the solution back together. This is particularly relevant to parallel computing. Finally, cyclic reduction has been used extensively in parallel computing because it allows the use of many processors simultaneously. This has been a very active area of research. [The *SCI*® indicates that this paper has been cited in more than 175 publications.]

Poisson's Equation Revisited

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As is often the case, this work began serendipitously. In the fall of 1968, Gene H. Golub, Stanford University, was visiting the Los Alamos National Laboratory (LANL) Computing Division to exchange information on current topics of research in numerical mathematics. Clair W. Nielson and I were employees of LANL at this time. During Golub's visit, Nielson stopped by my office to discuss a recently acquired software module for solving Poisson's equation on a rectangle, and Golub was invited to sit in on the discussion. Thus began a delightful research adventure. The software module that Nielson had obtained was from Oscar Buneman[1] at Stanford. It solved the classical five-point approximation to Poisson's equation at a speed and accuracy that far exceeded established techniques such as relaxation and alternating direction.[2] The documentation of the module was minuscule, and Nielson wanted to change the boundary conditions to handle a different coordinate frame and was seeking help in understanding Buneman's software. The documentation, combined with a rudimentary analysis of the source code, enabled us to mathematically describe the initial steps of the algorithm. One of the mysteries of the module was that it required that the number of rows and columns in the mesh be a power of two minus one. As inventor of the technique, Golub,[3] was well acquainted with cyclical odd-even reduction (CORF) and quickly saw a correlation between it and our analysis, including the mysterious power of two minus one. Nielson figured out the associated factorization scheme. At that point, we abandoned any further study of the software and proceeded to work out the CORF algorithm for several coordinate frames as discussed in section 3 of the final paper.

The three of us proceeded to carefully draft a manuscript of our findings. Since Golub had returned to Stanford, this took several weeks. At that point, I decided to program CORF to verify performance and accuracy. While the speed of CORF was similar to that of the Buneman module, to our amazement, CORF proved numerically unstable. I then developed an associated stability analysis as contained in section 10 of the final paper. Now we were really mystified! So, I went back to the Buneman software and extracted what is now known as the Buneman's fast Poisson solver, as discussed in section 11. We jointly developed the associated stability analysis as discussed in section 13.

At this point, we had two algorithms—CORF and Buneman's. Golub was absolutely convinced that the two were mathematically equivalent and, with great determination, he set for himself the task of uncovering that relationship. And he did so in the fall of 1969. The results were incorporated into section 11 of the final paper.

Many numerical experiments were performed to validate everything noted in the paper. Once the validation was complete, we began the process toward publication. So, over a period of about 18 months, with no small amount of mathematical sleuthhound-ing, we completed this *now-Classic* paper. During that 18 months, we were tempted on several occasions to publish intermediate results. However, we continued to hold out for a full understanding, and, in the end, we were especially pleased that we waited until we had a comprehensive report.

1. **Buneman O.** *A compact non-iterative Poisson solver.* Stanford, CA: Stanford University Institute for Plasma Research. 1969. Report 294.
2. **Varga R S**. *Matrix interative analysis.* Englewood Cliffs, NJ: Prentice-Hall, 1962. (Cited 1,650 times.)
3. **Gloub G H.** Unpublished thesis notes.

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