_This Week's Citation Classic ®

Hazell P B R. A linear alternative to quadratic and semivariance programming for farm planning under uncertainty. *Amer. J. Agr. Econ.* 53:53-62, 1971.
[School of Agriculture, Cornell University, Ithaca, NY]

Quadratic decision criteria for farm planning under risk are theoretically appealing but difficult to handle computationally. This paper develops a linear alternative that, while retaining most of the desired features of the quadratic models, can be readily solved by linear programming. [The *SCI®* and the *SSCI®* indicate that this paper has been cited in more than 100 publications, making it the most-cited article published in this journal.]

Making Risk Programming Accessible

Peter B.R. Hazell
The World Bank
International Bank for Reconstruction
and Development
International Development Association
Washington, DC 20433

The 1960s were an exciting era for production economists. Parallel advances in computers and mathematical programming were opening up new vistas for modeling and improving on real-world resource-allocation decisions.

As a Cornell graduate student in agricultural economics, I was particularly intrigued by the complexity of the decision problems confronting fresh market vegetable farms. Each year, these farmers face a bewildering set of choices about alternative crops and varieties to grow, when to plant them, how to grow them, and how, when, and where to market the harvested produce. Their choices must also be consistent with constraints on the land, glasshouse space, labor, machinery, and working capital available at different times, the need to rotate some crops in particular sequences to control pests and diseases, and the changing seasonal produce requirements of wholesalers. As if juggling all these variables was not enough, the business is also highly risky. Fresh vegetable prices are notoriously volatile and difficult to predict, and produce is easily ruined by a spell of unfavorable weather or by delays in transport and marketing.

I was challenged in the late 1960s to model the decision problem on the computer for a particular farmer to see if I could improve on his own heuristic decisions. H.M. Markowitz's pioneering, and subsequently Nobel Prize winning, work on portfolio investment theory provided a powerful new way of formulating the problem.[1] The initial problem had hundreds of activities and constraints, and, because of the quadratic form for income variance in the objective function, required a quadratic programming package. Therein lay one real problem, because available software could only cope with about 20 constraints.

I began to search for linear estimators of the variance that could be used to transform the problem into a standard linear programming model. Other researchers were pursuing the same approach for solving similar problems, but progress was stunted by the difficulty of linearizing all the covariances among activity returns while retaining the required concavity in the objective function.

My lucky break came when I realized that, since one usually estimated variance-covariance matrices with time-series data, one could build the sample data directly into the linear constraint set and have the model calculate total farm income for each sample year. It was then easy to collect the income variables in the objective function and, since only a single random variable was then involved, to use R.A. Fisher's mean absolute deviation estimator of the variance.[2]

My *Citation Classic* piece is the write-up of this linearization approach. Since the method involved the minimization of total absolute deviations, I coined "MOTAD." MOTAD has been widely used in risk programming applications, particularly in MS and PhD dissertations. MOTAD's computational advantages also facilitated the development of large-scale regional and sector planning models.[3] The method remains popular despite the development of modern nonlinear programming packages.

And what of my farmer? Did I improve on his planning capability? Not by much. Like the skilled pool player who deftly sinks balls without any inkling of the complex differential equations involved, my farmer proved to be very adept at his business. But at least I was able to assure him that he was not missing out on any easy gains.

1. Markowitz H M. *Portfolio selection: efficient diversification of investments.* New York: Wiley, 1959. (Cited 940 times.)
2. Fisher R A. A mathematical examination of the methods of determining the accuracy of an observation by the mean error, and by the mean square error. *Roy. Astron. Soc.* 80:758-69, 1920.
3. Hazell P B R & Norton R D. *Mathematical programming for economic analysis in agriculture.* New York: Macmillan, 1986. 400 p.
Received May 24, 1991