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Aris R. On the dispersion of a solute in a fluid flowing through a tube.
Proc. Roy. Soc. London Ser. A 235:67-77, 1956.
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Taylor showed that the effect of diffusion across a flow profile was to disperse a solute in the direction of flow with an apparent diffusion coefficient inversely proportional to the molecular diffusivity. This result is generalized by considering the growth of the *moments* of the distribution of the solute. [The *SCI*® indicates that this paper has been cited in more than 565 publications.]

Longitudinal Dispersion Due to Lateral Diffusion

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Sir Geoffrey Taylor's analysis of the dispersion of soluble matter in flow through a straight tube[1] could not have been published at a more opportune moment for me, as a raw young mathematician working at the Billingham Division of Imperial Chemical Industries. My assignment was to study the latest analytical tool, vapour phase chromatography, to see if a mathematical approach could improve its accuracy and efficiency. If there is a variation of flowrate across a tube, a small bolus of marked molecules will go down the tube as a compact bunch, since its members will dance back and forth between faster and slower streams and all experience the mean speed. If, on the other hand, their mobility across is restricted, those that happen to be near the center in the fast streams will stay there for a longer time and move farther, while those that happen to start in a slow stream will not move much at all. They will be much more spread out in the direction of flow. The combination of lateral diffusion and flow profile induces a longitudinal dispersion.

Taylor, using a simple physical argument and his insight into which terms were significant, had found the apparent diffusion coefficient for laminar flow in a tube of circular cross section to be $a^2U^2/48D$, where a is the radius, U the mean velocity, and D the molecular diffusion coefficient. Lacking Taylor's skill, I found his argument hard to follow, but chromatography had accustomed me to think in terms of the moments of a peak, and half the rate of growth of the variance is the apparent diffusion coefficient. Using this approach, I was able to remove Taylor's restrictions and get the general result of $(D+ka)^2U^2/D$, where k is a constant depending on the geometry of the tube and the flow. This combination, which allows for the diffusion in the direction of flow as well as across it, was confirmed by some careful experiments of Kenney[2] and others.

Whether any of this would have improved chromatography is hard to say, for I was scarcely started on it when I was told to design reactors and decided that the chopping and changing of industry was not for me. So the work was completed in the evenings and came out when I was first at Minnesota, being kindly communicated by Taylor to the *Proceedings of the Royal Society*. The moment approach caught on and was used in a number of chemical and civil engineering problems. It has been used by Brenner[3] in a very penetrating analysis of generalized dispersion and applied by him to many special cases. Some mathematical deficiencies of my paper, which did not come to light as I used only asymptotic forms, were kindly corrected by Barton[4] in 1983. An extension to oscillating flow[5] that I made in 1960 has quite recently been applied by Leighton and McCready[6] to the enhancement of separation by liquid membranes in a device soon to be patented.

The popularity of the paper has much to do with the happenstance of timing. The moment approach was new to me because I didn't know the literature well. And the generalizations were matters of detail once Taylor's basic insight was grasped. Some have been kind enough to use the term Taylor-Aris dispersion. I take it as a very great honor to be so associated, though the honor is really to Taylor, whose name could be properly attached to so many things that it is useful to hyphenate him with an ordinary mortal.

1. Taylor G I. The dispersion of soluble matter in a solvent flowing through a tube. *Proc. Roy. Soc. London Ser. A* 219:186-203, 1953. (Cited 135 times.)
2. Evans E V & Kenney C N. Gaseous dispersion in laminar flow through a circular tube. *Proc. Roy. Soc. London Ser. A* 284:540-50, 1965. (Cited 35 times.)
3. Brenner H. A general theory of Taylor dispersion phenomena. *Physicochem. Hydrodyn.* 1:91-123, 1980. (Cited 45 times.)
4. Barton N G. On the method of moments for solute dispersion. *J. Fluid Mech.* 126:205-18, 1983. (Cited 10 times.)
5. Aris R. On the dispersion of a solute in pulsating flow through a tube. *Proc. Roy. Soc. London Ser. A* 259:370-6, 1960. (Cited 35 times.)
6. Leighton D T & McCready M J. Shear enhanced transport in oscillatory liquid membranes. *AIChE J.—Amer. Inst. Chem. Eng.* 34:1709-12, 1988.

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