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Coleman B D & Noll W. The thermodynamics of elastic materials with heat conduction and viscosity. *Arch. Ration. Mech. Anal.* 13:167-78, 1963.
[Mellon Institute; and Department of Mathematics, Carnegie Institute of Technology, Pittsburgh, PA]; and
Coleman B D. Thermodynamics of materials with memory. *Arch. Ration. Mech. Anal.* 17:1-46, 1964.
[Mellon Institute, Pittsburgh, PA]

In the first paper, the authors present a method for determining the restrictions that the second law of thermodynamics places on constitutive relations. In the second paper, the method is applied to a general theory of materials with gradually fading memory. [The SCI® indicates that these papers have been cited in over 295 and 330 publications, respectively.]

Thermodynamics and Constitutive Relations for Materials with Memory

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In the early 1950s, for graduate courses in physical chemistry at Yale University, I read the then standard texts on thermodynamics and selections from the works of Gibbs. I recall being awed by the generality of the subject; it appeared to have deep implications in every branch of macroscopic physics; but I also recall being uneasy about the completeness or clarity (I was not sure which) of its principles. About five years later, after discussions with Walter Noll, I recognized the source of my uneasiness. If one truly understood the principles of thermodynamics, one should be able to give precise mathematical procedures for finding not just conditions for the stability of equilibrium, but also the restrictions that the second law of thermodynamics imposes on the equations (called "constitutive relations") that characterize the type of material under consideration, and such procedures were not available. Of particular interest to me were constitutive relations that express the stress tensor as a function of the history (i.e., of both the present and past values) of the strain and temperature.

In the years 1958-1961, I worked with Noll on several problems in the theory of viscoelastic materials, or, more generally, materials with gradually fading memory. One of our early joint papers[1] on such materials was the subject of a commentary for this column some years ago; another[2] recently was suggested by ISI® editors for a commentary. A primary goal of my own work on the theory of such materials was the attainment of an understanding of the restrictions that the second law places on their behavior. By 1961 I felt that I knew, on the basis of physical reasoning, what the restrictions were, but I had no framework in which to give a mathematical justification of my ideas.

It was not until the fall of 1962 that Noll and I found, after several false starts, the methods presented in the paper of 1963 cited above for deriving conditions necessary and sufficient for a given class of constitutive assumptions to be compatible with the second law. In that paper we applied the method to a class of materials for which (in contrast to materials with gradually fading memory) the implications of the second law were already considered self-evident. In an article[3] published in the next issue of the same journal, Victor Mizel and I gave the method an equivalent but more transparent form and showed that the "principle of equipresence" that Clifford Truesdell[4,5] had proposed as a guide for the formulation of constitutive assumptions can be implemented when thermodynamical variables and principles are taken into account.

Once the method was in hand, I believed I could present a convincing justification of what I felt was the complete set of thermodynamical restrictions on constitutive relations when gradually fading memory is present, but I soon realized that I had to learn more functional analysis before I could present arguments that met the current standards of mathematical rigor. Another year had to pass before the second paper cited above could be written. As with the paper of 1963, shortly after the paper of 1964 went to press, a way was found to clarify its principal concepts and extend its conclusions, and I soon sent off a follow-up paper[6] for the same volume of the journal. The formalism of that follow-up paper is that employed in nearly all subsequent applications of the theory presented in the 1964 paper.

1. **Coleman B D & Noll W.** An approximation theorem for functionals, with applications in continuum mechanics. *Arch. Ration. Mech. Anal.* 6:355-70, 1960. (Cited 350 times.) [See also: **Coleman B D.** Citation Classic. *Current Contents/Engineering, Technology & Applied Sciences* and *CC/Physical, Chemical & Earth Sciences* 13 January 1986. p. 16.]
2. ------- . Foundations of linear viscoelasticity. *Rev. Mod. Phys.* 33:239-49, 1961. (Cited 310 times.)
3. **Coleman B D & Mizel V J.** Thermodynamics and departures from Fourier's law of heat conduction. *Arch. Ration. Mech. Anal.* 13:245-61, 1963. (Cited 55 times.)
4. **Truesdell C.** A new definition of a fluid, II. The Maxwellian fluid. *J. Math. Pure Appl.* 30:111-55, 1951.
5. **Truesdell C & Toupin R A.** The classical field theories. (Flügge S, ed.) *Encyclopedia of physics.* Berlin, FRG: Springer, 1960. Vol. III/1. p. 703-4.
6. **Coleman B D.** On thermodynamics, strain impulses, and viscoelasticity. *Arch. Ration. Mech. Anal.* 17:230-54, 1964. (Cited 150 times.)

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