This Week's Citation Classic®

The mathematical singularities of catastrophe theory provide a classification of the caustics (focal envelopes) of families of rays (e.g., in optics and quantum mechanics). In the short-wave limit, caustics dominate wavefields. The classification also gives a description of the complicated interference patterns that decorate caustics. [The *SCI®* indicates that this paper has been cited in over 135 publications.]

Catastrophes and Waves

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January 4, 1990

From earlier studies of the short-wave limit of optics or quantum mechanics,[1] it had gradually emerged that in the families of rays corresponding to a wavefield the most important features are their *caustics*. These are the focal singularities of the family, that is, envelopes that each ray touches. This understanding was obtained by accumulation of particular cases in potential scattering and electron microscopy. For each case, it was possible to determine in detail how the singularities are softened by diffraction. It seemed that the topology of the singularity has a crucial effect on the associated diffraction, but before 1974 I had only vague ideas about how to formulate this notion.

The crucial step towards the construction of a complete theory of the short-wave limit was reading R. Thom's book on catastrophes.[2] I found this work mysterious but was greatly helped by an unpublished exegesis by Zeeman. Thom discovered that singularities of certain smooth mappings (derived from gradients) can

be classified. The classification (later enormously extended by Arnold) was by codimension, which is the number of parameters that must typically be explored to find the singularity. Thom realized that his classification described optical caustics (via Fermat's principle, according to which a family of light rays is a gradient map generated by the travel time function).

In catastrophe theory an important idea is that the caustics it classifies are structurally stable, that is unaltered (apart from being deformed) by perturbation. Therefore, it can describe caustics as they occur in nature, without the symmetry required for optical imaging.

Another discovery was that as well as describing the caustics, the catastrophes could be employed to construct diffraction integrals for the wave patterns that decorate them. Of this hierarchy of "diffraction catastrophes," the first two had been studied before (by Airy in 1838 and Pearcey in 1946). The patterns are intricate and beautiful and can be "stretched" to provide quantitatively accurate approximations to wavefields, uniformly valid near and far from caustics.

My main interest at that time was short waves in quantum mechanics, and the first application of the new ideas was to the scattering of atoms by solid surfaces.[3] However, two circumstances combined to convince me that the main source of novel applications of the catastrophe classification was likely to be optics. First was an interest in the curious distortions of lights seen through irregular water-droplet "lenses." Second was a chance remark by Dr. J.A. Barker about strange patterns of sunlight in rippled bathroom-window glass.

Catastrophes turned out[4] to play a "mesoscopic" role in wave physics, with the caustic singularities acting like "atoms of form." They organize the fine details of "microscopic" interference patterns and are elements of "macroscopic" caustic networks such as those on the bottoms of swimming pools (formed by sunlight refracted by the wavy surface). There have been many applications of these ideas, for example, to the near-field of liquid drop lenses[5] (where high catastrophes are involved) and in seismology.[6]

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2. Thom R. Stabilité structurelle et morphogenèse (Structural stability and morphogenesis). New York: Benjamin, 1972. 362 p. (Cited 380 times.)
3. Berry M V. Cusped rainbows and incoherence effects in the rippling-mirror model for particle scattering from surfaces. *J. Phys.—A—Math. Gen.* 8:566-84, 1975. (Cited 55 times.)
4. Berry M V & Upstill C. Catastrophe optics: morphologies of caustics and their diffraction patterns. *Prog. Optics* 18:257-346, 1980. (Cited 65 times.)
5. Nye J F. The catastrophe optics of liquid drop lenses. *Proc. Roy. Soc. London Ser. A* 403:1-26, 1986. (Cited 5 times.)
6. ——————. Caustics in seismology. *Geophys. J. Roy. Astron. Soc.* 83:477-85, 1985. (Cited 5 times.)

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