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Bilby B A, Cottrell A H & Swinden K H. The spread of plastic yield from a notch. *Proc. Roy. Soc. London Ser. A* 272:304-14, 1963.
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In strong metals brittle fracture starts from a notch only after plastic deformation has occurred locally there. The calculated length of plastic zone, to accommodate this deformation, explains why large samples can be weak and brittle when small ones are strong and ductile. [The *SCI*® indicates that this paper has been cited in over 460 publications.]

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Why Some Brittle Solids Are Strong

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A glass windscreen is easily shattered by a small pebble. A steel cold chisel withstands far rougher treatment, yet appears just as brittle. Ordinary structural steel is also brittle, but only if cold (usually below 0° C), in big pieces, and with notches some millimetres or centimetres deep. Small pieces remain ductile. The sharpness at the end of the notch is unimportant, below a certain limit.

I thought in the late 1950s that the explanation lay in local ductility at the end of the notch. This might be sufficient to blunt the notch, wiping out the effect of extreme sharpness that causes scratched glass to break easily. After some plastic stretching, the material there could break and the ensuing crack, gathering speed, could then travel as a brittle fracture. This local plastic stretching would need to be accommodated by a plastic zone spreading into more distant regions. If the piece were too small, the zone would run out to the far side and the material could then not break below its general yield stress. If the piece were big, however, often many centimetres in structural steel, it could contain the plastic zone and thus break below its general yield stress, appearing weak and brittle.

To develop this idea, an elastic-plastic theory was needed. Only the theory of dislocations met this need in the 1950s. I visualized the interior of the notch as containing, formally, a pileup of dislocations, pressed up against its sharp end by the applied stress, and some of these dislocations then "leaking through" into the material ahead, to form the plastic zone. I soon wrote down the mathematical equations representing this situation and equally quickly realized that I could not solve them.

And so I approached my old friend and research collaborator, B.A. Bilby, a much better mathematician, at the University of Sheffield. He had a bright research student, K.H. Swinden, and they found an analytical solution. Working by correspondence, between Sheffield and Cambridge, we explored its physical consequences, which confirmed my intuitive ideas. It could all have been published in about 1960, but we got drawn into other things, and it was delayed until 1963. I did however summarize the work in an earlier paper.[1]

Two aspects of our paper have found practical applications. The concept of a critical plastic stretch, before the notch breaks, which was also introduced independently by A.A. Wells[2] as the "crack opening displacement," has become a popular design criterion in the fracture mechanics of large steel structures.[3] Secondly, the "log.sec." relation has proved useful for estimating the lengths of plastic zones that need to span the cross section, for strong fractures.[4-6]

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