

**Murman E M & Cole J D.** Calculation of plane steady transonic flows.  
*AIAA J.* 9:114-21, 1971.  
[Boeing Scientific Research Laboratories, Seattle, WA]

In this paper a solution procedure was introduced for a mixed (elliptic-hyperbolic) partial differential equation (PDE) modeling transonic flow of a compressible gas. A boundary value problem was also formulated that included a far field solution for an airfoil. We solved the nonlinear PDE using a simple combination of elliptic and hyperbolic finite difference formulas. Computed results demonstrated that the method was successful. [The *SC7*® indicates that this paper has been cited in over 170 publications.]

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August 31, 1987

By the mid-1960s the performance of transport and military aircraft, helicopter rotors, and axial flow turbines and compressors was limited by transonic flow phenomena. The simultaneous occurrence of subsonic and supersonic flow with embedded shock waves could not be modeled by existing analytical or computational procedures. Engineering designs for these devices required lengthy, expensive wind-tunnel tests. The introduction of supercritical airfoil sections by Richard Whitcomb of the National Aeronautics and Space Administration in Langley indicated that significant performance improvements could be achieved. But there was no way to explore these designs using aerodynamic theory.

We were brought together in the fall of 1968 by Arnold Goldburg, director of the Flight Sciences Laboratory of the Boeing Company. E.M. Murman had been studying numerical methods for solving partial differential equations (PDEs) after receiving his PhD in 1967. J.D. Cole was on a year's leave of absence from the California Institute of Technology, with

plans to revisit transonic aerodynamic theory. Goldburg suggested we collaborate and try to develop a solution procedure for transonic flows. The availability of digital computers offered a new approach to this problem.

The challenge was to find a way to integrate a PDE of mixed (elliptic and hyperbolic) type that contained embedded discontinuities (shock waves). Cole felt that a computational approach might be possible and that it should be matched to an outer analytical solution in the far field. He formulated such a boundary value problem, using asymptotic expansions. But there were no available finite difference-type methods for mixed equations.

The breakthrough for the numerical algorithm came in early 1969 during an afternoon brainstorming session between the two of us. After trying to extend elliptic equation algorithms to mixed problems and failing, we came up with the idea of combining the two. A simple test was introduced to determine at each mesh point, as the iterative solution converged, whether or not an elliptic or hyperbolic difference formula should be used. The remaining details of the method fell into place with a little more effort, and a paper was presented at the January 1970 American Institute of Aeronautics and Astronautics Aerospace Sciences Meeting in New York.

This paper has probably been cited so often because it presented the right idea at the right time. The computational method was simple and efficient. Within a year or two of its publication, computer codes were written that gave satisfactory agreement with experimental tests for two- and three-dimensional wings. Aerodynamists finally had an analysis method for transonic flow, and the terrific demand for such a tool fueled a rapid period of development.

More modern analysis methods for the Euler and Navier-Stokes equations have since replaced the simpler approaches of this paper. However, at a time when computers were not powerful and plentiful and there was little understanding of the power of computational analysis, this method convincingly demonstrated a new direction. The method still serves as a way to bridge classical aerodynamic theory based on potential flow with modern computational methods.

[For a recent paper in this field see reference 1.]

1. Jameson A. Successes and challenges in computation aerodynamics. *Proceedings of the AIAA 8th Computational Fluid Dynamics Conference*. New York: AIAA, 1987. p. 1-35. AIAA 87-1184.

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