This Week's Citation Classic®

Cliff N. Orthogonal rotation to congruence. *Psychometrika* 31:33-42, 1966.
[University of Southern California, Los Angeles, CA]

A method is derived that rotates to maximum similarity the factor loadings from two factor analyses or the dimension loadings from two multidimensional scaling studies, or that orthogonally rotates an obtained factor matrix to maximum fit to a theoretical one. It is noted that the transformation matrix in the second case is the product of the two in the first. [The *SCI*® and *SSCI*® indicate that this paper has been cited in over 155 publications.]

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In the early 1960s I worked with data problems in which I compared the solutions to different multidimensional scaling analyses. I derived the different sets under different experimental conditions because I was investigating the possibility that the experimental conditions altered the configuration of points. However, the coordinates might have looked different simply because one matrix was a rotation of the other. Therefore, I wanted a way to rotate sets of coordinates so that they were as nearly congruent as possible. I called this problem Case I in the paper discussed here, and I found a solution that rotated each matrix so that the points in each were as close as possible to the same positions on the two sets of axes. I applied this procedure to several such sets of points.

At about the same time, a colleague, J.P. Guilford, approached me with a slightly different problem. He wanted to take a single set of factor loadings that had an arbitrary orientation in space and rotate them to come as close as possible to a *theoretical* position.

After mulling it over I realized that the two problems were the same. Suppose you spread the fingers of both of your hands and hold the hands in front of you, with the fingers of one hand pointing up and those of the other pointing down. In the first problem you rotate both wrists in opposite directions until the two sets of fingers point in the same direction. In the second problem you leave one hand, say the left, where it is and rotate the right hand until its fingers are pointing in the same direction. The position of the two hands *relative to each other* is the same in both cases. In a mathematical sense, the transformation in the second case is the product of the two transformations in the first—thus the connection between the two solutions.

This procedure suited Guilford's need very well, and he and his colleagues used it in many subsequent studies.[1,2] My coworkers and I also used it in an extensive study of sampling errors in factor analysis.[3]

I prepared a paper for publication that described the two problems and their solution, and it was accepted by *Psychometrika*. Coincidentally, another paper[4] that solved the second problem appeared in the same journal issue, the editors apparently feeling that it was appropriate to publish both since the work was done independently and the approaches were different.

Citations to my paper seem to be of two sorts. One sort applies the methodology, as in Guilford's work. Perhaps the paper is popular because it provides a relatively simple solution to two relatively common problems. Quick perusal of recent *SSCI* citations suggests, however, that in recent years this type of citation has not been so common, perhaps because of the obsolescence and relative unavailability of appropriate software.

The other kind of citation, which has been more frequent in recent years, is by articles containing related methodology.[5] The fact that these are still relatively common suggests that this paper constitutes part of a methodological context that is of continued relevance.

1. **Guilford J P & Hoepfner R.** *The analysis of intelligence.* New York: McGraw-Hill, 1971. 514 p. (Cited 140 times.)
2. **Guilford J P.** A sixty-year perspective on psychological measurement. *Appl. Psychol. Meas.* 9:341-9, 1985.
3. **Cliff N & Pennel R.** The influences of communality, factor strength and loading size on the sampling characteristics of factor loadings. *Psychometrika* 32:309-26, 1967.
4. **Schönemann P.** A generalized solution of the orthogonal problem. *Psychometrika* 31:1-10, 1966. (Cited 165 times.)
5. **Ramsay J O, ten BerGe J & Styan G P H.** Matrix correlation. *Psychometrika* 49:403-23, 1984.

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