

Lawler E L & Wood D E. Branch-and-bound methods: a survey.

*Oper. Res.* 14:699-719, 1966.

[University of Michigan, Ann Arbor, MI]

Combinatorial optimization problems generally cannot be solved by straightforward enumeration of feasible solutions because there are simply too many of them. However, a method of implicit enumeration called branch-and-bound has been applied successfully to a variety of problems. [The *SCI*® indicates that this paper has been cited in over 165 publications.]

Eugene L. Lawler  
Department of Electrical Engineering  
and Computer Sciences  
College of Engineering  
University of California  
Berkeley, CA 94720

September 24, 1986

Typical problems in combinatorial optimization are to find the best sequence for performing a set of tasks, the best routing for a set of school buses, or the best choice of locations for warehouses in a distribution system. In each case it is necessary to find the best solution from among a very large number of possibilities.

Actually, these problems are more difficult than looking for a needle in a haystack. When you have found a needle, at least you know you are done. These problems are more like searching for the smallest needle in a very large pile of needles. When you have found a very small needle, how do you know that it is the smallest, without comparing it with all the others? Even the fastest computer may not be capable of doing this, because of the enormously large number of needles.

As a graduate student at the Harvard Computation Laboratory, I learned of an interesting computational technique that Willard Eastman had used to solve moderately large instances of the notorious "traveling salesman problem." Roughly speaking, his approach was as follows: "branch" to divide the pile of needles into two or more small piles. Then "bound" by computing a lower bound on the size of the smallest needle in each of the piles. Repeat this procedure until you find a needle whose size is at least as small as the lower bound for each of the existing piles.

In my PhD thesis I applied Eastman's ideas to a generalization of the "traveling salesman problem" known as the "quadratic assignment problem." At the time I wrote my thesis, I was already aware that the same approach had been discovered independently by others. But it was only a bit later that I came to appreciate how widely the same ideas had been applied. I thought it might be useful to write a survey paper and enlisted David Wood, a graduate student at the University of Michigan, as a coauthor. In our paper we showed how "implicit enumeration," "séparation et évaluation progressives," and other methods all amounted to "branch-and-bound," the descriptive term coined by Little et al.<sup>1</sup>

Great progress has been recorded in the field of combinatorial optimization during the past 20 years. Yet branch-and-bound methods remain an indispensable part of the repertoire of the algorithm designer. The large number of times our paper has been cited is surely due to the fact that authors could avoid explaining things from scratch by simply saying, "We propose a branch-and-bound approach, as described by Lawler and Wood..." or words to that effect. The state of the art of branch-and-bound, with respect to the "traveling salesman problem," is described by Balas and Toth.<sup>2</sup>

1. Little J D C, Murty K G, Sweeney D W & Karel C. An algorithm for the Traveling Salesman Problem.

*Oper. Res.* 11:972-89, 1963. (Cited 165 times.)

2. Balas E & Toth P. Branch and bound methods. (Lawler E L, Lenstra J K, Rinnooy Kan A H G & Shmoys D B, eds.)

*The Traveling Salesman Problem*. New York: Wiley, 1985. p. 361-401.