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Coleman B D & Noll W. An approximation theorem for functionals, with applications in continuum mechanics. *Arch. Ration. Mech. Anal.* **6**:355-70, 1960.
[Mellon Institute and Mathematics Department, Carnegie Institute of Technology, Pittsburgh, PA]

The authors construct a general theory of "fading memory" and find the circumstances under which functional relations covered by their theory are approximated by relations that occur in classical theories of viscous damping and rate-dependent stress. [The *SCI*® indicates that this paper has been cited in over 310 publications since 1960—one of the most-cited papers for this journal.]

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In the late 1950s Walter Noll and I were working together on a new class of problems in continuum mechanics: the calculation of flow fields by semi-inverse methods employing the most general constitutive relations compatible with the assumed symmetry group of the material. We were interested in materials for which the stress depends upon the history of the strain. When we attempted to formulate a theory of the thermodynamics of such materials, it became clear that something new was needed beyond a consideration of symmetry groups. We needed a *general* way to render mathematical the idea that a material that appears to be elastic in very fast and very slow deformations will often, when subjected to deformation histories between these two extremes, appear to be viscoelastic with a memory that fades gradually in time. Various special constitutive relations with these properties had been formulated and studied by Maxwell, Boltzmann, Volterra, and also numerous modern workers in continuum mechanics and rheology. We sought a formulation of this "principle of fading memory" that would rest upon general concepts from topology and functional analysis.

While seeking to formulate a certain relation in thermodynamics, we came up with a smoothness assumption and a class of Banach function spaces that appeared to supply the sought-after formulation of the "fading memory principle." It was clear that the class of constitutive relations that obey this smoothness assumption contains as special cases those having the structure of Boltzmann's linear theory of viscoelasticity. Rate-type constitutive relations, such as the constitutive equation of a Navier-Stokes fluid (sometimes called a "Newtonian fluid"), do not occur as special cases of the fading memory principle, but it appeared that they should be asymptotic forms, valid in the limit of slow motions. It required work, however, to make this idea precise, and apparently nontrivial theorems needed to be proved. The 1960 paper gives the result of that work; the second theorem of the paper, the "retardation theorem," makes precise the asymptotic status of rate-type materials within the class of materials with gradually fading memory.

The problems addressed in the paper were formulated using contemporary concepts of analysis that, although familiar to mathematicians, were new to many physicists and were considered unnecessarily abstract by several of those whose experience had been confined to the study of the special constitutive relations of what may be called "classical rheology." It appears, however, that time has shown that our early quest for generality and rigor in the theory of constitutive equations was not without practical value. Many subsequent papers[1,2] have invoked our "retardation theorem" to justify "slow flow" approximations made to facilitate the solution of hydrodynamical problems for nonlinear viscoelastic fluids. The theory of fading memory introduced here was later applied to obtain information about the existence, uniqueness, and stability of solutions of functional-differential equations with long-range memory, and to obtain formulae for the velocity and rate of growth of singular surfaces in dissipative media.

As a personal note I should mention that, at the time this paper was written, Walter Noll was a professor of mathematics at the Carnegie Institute of Technology, and I was a senior fellow (albeit more junior than senior) of the Mellon Institute. Although Noll was educated as a mathematician, my doctorate was in physical chemistry, and if it were not for the encouragement of the administrative head of the "Mellon Institute Polymer Group," Thomas G Fox, and the friendly patience of my colleagues at the two institutes, which later merged to form Carnegie-Mellon University, I should never have learned the functional analysis required for the type of work described here.

1. **Joseph D D.** *Stability of fluid motions II.* Berlin: Springer-Verlag, 1976. p. 200-61.
2. **Lodge A S, Renardy M & Nohel J A.** *Viscoelasticity and rheology.* Orlando, FL: Academic Press, 1985.

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