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Smith A M O & Clutter D W. Solution of the incompressible laminar boundary-layer equations. *AIAA J.* 1:2062-71, 1963.
[Douglas Aircraft Co., Inc., Long Beach, CA]

A method is presented for solving the complete incompressible laminar boundary-layer equations, both two-dimensional and axisymmetric, in essentially full generality and with speed. Previous methods all had some kind of major restriction—either speed or important problems that could not be solved. [The *Science Citation Index*® indicates that this paper has been cited in over 75 published papers, making it one of the 10 most-cited articles published in this journal.]

A.M.O. Smith
2245 Ashbourne Drive
San Marino, CA 91108

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"At the time this paper was written, I was in charge of aerodynamic research at the Douglas Aircraft Co. I had had good luck in solving the so-called Neumann problem of inviscid, incompressible potential flow for truly arbitrary shapes. After getting over this hurdle, it was logical that I tackle the equivalent viscous problem because the real flow over any body is a mixture of viscous and inviscid types. I felt that accurate solutions of the laminar boundary layer should be useful and, if successful, might lead to extended forms of the equations—compressible, etc.

"Therefore, around 1958, with the help of Darwin Clutter, I began examining all the existing methods of solution to try to find something that would be truly general. We gradually realized we had found it. The equations involved some transformation but nothing tricky. We began calculation work by trying to solve the simpler similar form of the equations. Considerable study had been made of various methods of solution, and we decided Picard's iterative method[1,2] looked very good. The full partial differential equation was to be solved by solving ordinary differential equations at a number of stations along a body. Picard's method worked like a charm, and we could not see that adding the small non-similar corrective terms should change things. Therefore, at that time, we sought support from the new US Navy Bureau of Weapons and obtained it.

"Then our troubles began. Picard's method now did not converge. Try as we might, we could not make it converge, and, being under contract, time was running out. In desperation, we switched to an old 'shooting method' that I had used some 10 years before. It worked perfectly. So our carefully studied method went awry and the successful method was a spur-of-the-moment choice.

"The entire development was pulled together in reference 3. I never got any awards for the work directly, but indirectly and in line with my stated goals, I was co-winner of the Casey Baldwin Award of the Canadian Aeronautics and Space Institute for a paper on combining potential flow and boundary-layer calculations.[4] The paper was important because I believe it was the first to present a general method that could give accurate answers for any problem that might arise. I think the method was the first that might be called a production code. Also, it presented solutions of problems that had not been solved before. But in examining the citations, I see no particular pattern running through them. The paper is cited for all kinds of reasons—from nothing more than that it is there, all the way to using its methods in various problems. My ultimate goal was calculation of turbulent boundary layers. In them, the eddy viscosity is unknown ahead of time, and this method, while successful, proved very slow because a double iteration was involved. We switched to ordinary finite difference methods that have mostly replaced this older method,[5] but this original method still remains very simple conceptually and very powerful."

1. **Picard E.** Mémoire sur la théorie des équations aux dérivées partielles et la méthode des approximations successives. *J. Math. Pure. Appl.* 6(Ser. 4):145-210, 1890.
2. **Milne W E.** *Numerical solution of different equations.* New York: Wiley, 1953. 275 p.
3. **Jaffe N A & Smith A M O.** Calculation of laminar boundary layers by means of a differential-difference method. *Prog. Aerosp. Sci.* 12:49-212, 1972.
4. **Bauer A B, Smith A M O & Hess J L.** Potential flow and boundary-layer theory as design tools in aerodynamics. *Can. Aeronaut. Space J.* 16:53-69, 1970.
5. **Cebeci T & Smith A M O.** *Analysis of turbulent boundary layers.* New York: Academic Press, 1974. 404 p. (Cited 115 times.)

