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Shapiro S S & Wilk M B. An analysis of variance test for normality (complete samples). *Biometrika* **52**:591-611, 1965.
[General Electric Co., and Bell Telephone Laboratories, Inc., Murray Hill, NJ]

This paper describes a procedure for testing the assumption that a random sample was drawn from a normal distribution. A ratio of two estimates of the population variance is used. The ratio is origin- and scale-invariant and is thus appropriate as a test of a composite hypothesis. [The *Science Citation Index®* (*SCI®*) and the *Social Sciences Citation Index®* (*SSCI®*) indicate that this paper has been cited in over 320 publications since 1965.]

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This paper was based on my PhD thesis, which was directed by Martin Wilk at Rutgers—the State University, New Brunswick, New Jersey. The work arose from an idea of Wilk that the information that is exhibited in a normal probability plot, a technique that is used to assess graphically whether the configuration of a body of data is consonant with the assumption that it is a random sample drawn from a normal population, could be quantitized and analyzed by standard, probabilistic statistical procedures. After investigating several possible approaches, the analysis of variance (ANOVA) technique seemed most promising. The ANOVA technique compares two estimators of the population variance, one that is valid whether or not the hypothesis (in this case normality) is correct and a second that is a valid estimator only if the hypothesis is true.

Once a procedure was identified, the next steps were to determine its statistical properties, obtain its distribution so that one could determine critical values for the test of normality, and assess its ability to detect nonnormality of the sample configuration (power of the test). Some statistical properties could be determined analytically; however, the statistical distribution and power of the test could only be estimated via simulation. Today, with high-speed computers, this would not be a big problem, but in 1960 a large simulation study was a major undertaking. Rutgers had an IBM 650 computer with a 2,000-word drum and punch-card input, and by today's standards it was extremely slow. After many overnight attempts at feeding cards into the machine and having problems with blown fuses and cards jamming the feeder, it looked like the simulation would never be completed. Fortunately, we were able to use a "new" IBM 704 computer at Bell Telephone Laboratories. This new machine had a tape feed and was a factor of 10 faster than the 650. The results of the power study were extremely good, and comparisons with the then-available procedures indicated that the W Test (which it was subsequently named) was substantially more powerful as an omnibus test procedure.

The test procedure represented a new approach to testing distribution assumptions and has stimulated researchers in new directions so that a large number of procedures have emerged since that time. To date, the W Test is still one of the most powerful of the omnibus tests and hence is used as a standard of comparison for new procedures. Thus it has been cited many times. In addition, it has been extended to other distributions such as the exponential[1] and multivariate normal.[2] A recent review paper[3] describes a number of these new procedures.

1. **Shapiro S S & Wilk M B.** An analysis of variance test for the exponential distribution (complete samples). *Technometrics* **14**:355-70, 1972.
2. **Malkovich J F & Afifi A A.** On tests of multivariate normality. *J. Amer. Statist. Assn.* **68**:176-9, 1973.
3. **Shapiro S S & Brain C W.** Recommended distributional testing procedures. *Amer. J. Math. Sci.* **2**:175-222, 1982.