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't Hooft G & Veltman M. Regularization and renormalization of gauge fields. *Nucl. Phys. B* **44**:189-213, 1972.
[Institute for Theoretical Physics, University of Utrecht, the Netherlands]

A new regularization and renormalization procedure is presented based on extension of a gauge field theory toward a complex number of dimensions. The method respects unitarity, causality, and gauge-invariance in the form of Ward identities. The paper discusses in particular how to disentangle overlapping divergences. [The *SCI*® indicates that this paper has been cited in over 790 publications since 1972.]

Gerard 't Hooft
Institute for Theoretical Physics
University of Utrecht
3508 TA Utrecht
The Netherlands

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"The swing in favor of the conventional quantum field theories in spite of the occurrence of divergent integrals came in the beginning of the 1970s, when the use of 'local non-Abelian gauge invariance' became respectable. This notion had been introduced as far back as 1954 by C.N. Yang and his student R. Mills[1] but two further developments turned out to be crucial. One was the introduction of 'spontaneous local symmetry breaking,' by P.W. Higgs[2-4] *et al.* The other was that we learned how to deal with the divergent integrals in these 'gauge field theories.' First, it was understood how to formulate these gauge theories in such a way that divergences do not proliferate at higher orders ('renormalizability') and, secondly, it was understood how the surviving divergent expressions could be combined to yield meaningful results.[5]

"However, it was not totally clear that the proposed methods would lead to unique, unambiguous calculations. To show this, it would be most desirable to introduce a so-called 'regulator,' replacing divergent integrals temporarily with convergent ones. The big problem was to do this without spoiling local gauge-invariance and other desired properties of a theory. One trick had been discovered before: our theories could be formulated in five rather than four space-time dimensions. The fifth dimension could be used to regulate diagrams with one loop.[5] Could we extend this trick to disentangle diagrams with more than one closed loop using six or more dimensions? The correct answer turned out to be to use $4+\varepsilon$ dimensions, by first carefully elaborating the mathematical definition of a theory in non-integer dimensions. Then one takes the limit $\varepsilon \rightarrow 0$. This was inspired by the observation that typical expressions for Feynman integrals contain functions such as $\Gamma(\alpha - \frac{1}{2}n)$, where α is a (positive or negative) integer; n is the number of dimensions; $\Gamma(x)$ is only infinite if $x=0, -1$, etc.; and α could not be changed because of gauge-invariance. So we had to take n to be a non-integer. It worked. Special care had to be taken to ensure that subtraction of the poles in the form of $1/\varepsilon^k$ could be done in the form of local counter terms, even in the case of overlapping divergences.

"The method of dimensional regularization, as it is now called, was discovered independently by C.G. Bollini and J.J. Giambiagi,[6] and by J.F. Ashmore.[7] In our paper, we worked out explicitly the reasons for gauge invariance and infinity cancellation in higher loops.

"As it turned out, the method has a much wider use than to provide a convergence proof in gauge theories. It is a powerful and quick way to compute divergent diagrams and to label the infinities in the form of poles. Also, the behavior of a field theory under scaling (asymptotic freedom) can most easily be computed this way. For a review, see reference 8."

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3. ---------------. Broken symmetries and the masses of gauge bosons. *Phys. Rev. Lett.* **13**:508-9, 1964. (Cited 240 times.)
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5. **'t Hooft G.** Renormalizable Lagrangians for massive Yang-Mills fields. *Nucl. Phys. B* **35**:167-88, 1971. (Cited 505 times.)
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8. **Narison S.** Techniques of dimensional regularization and the two-point functions of QCD and QED. *Phys. Rep.—Rev. Sect. Phys. Lett.* **84**:263-399, 1982.