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This Week's Citation Classic

Warburton G B. The vibration of rectangular plates.
Proc. Inst. Mech. Eng. **168**:371-81, 1954.
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The Rayleigh method is used to derive a simple approximate frequency expression for the modes of vibration in flexure of rectangular plates with various boundary conditions. This expression is of use to designers who require rapid estimates for natural frequencies. [The *SCI*® indicates that this paper has been cited over 100 times since publication, making it the most-cited paper ever published in this journal.]

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"Rectangular plates may be regarded as a first approximation to wings and blades and occur as panels in many forms of engineering structures. Thus a knowledge of their natural frequencies is often of considerable importance at the design stage. Thirty years ago such knowledge was limited. Most results had been obtained by the Rayleigh-Ritz method, which was then the most useful approximate technique but required considerable computation—tedious computation then, although routine today. Results were limited to specific sets of boundary conditions, modes, and aspect ratios; if the designer required natural frequencies for other conditions, he was left with a considerable task. My starting point in this work was to attempt to find a simple frequency expression, which could be used in the design office, even using the slide rule that was then conventional, and would give natural frequencies of acceptable accuracy. The obvious starting point was to use the Rayleigh quotient, being encouraged by the simple treatment of related problems by Rayleigh in his book *The Theory of Sound*.[1] (Rayleigh's contributions to modern vibration analysis, through his use of physical insight, simple concepts, and energy principles, were discussed in my recent Rayleigh Medal Lecture to the Institute of Acoustics.[2]) In this manner, a simple and versatile frequency expression was obtained; its accuracy was assessed by comparison with published theoretical and experimental results. It is interesting to note that the appreciable volume of results on natural frequencies of rectangular plates generated in the last 30 years has mainly confirmed these predictions of accuracy.[3]

"From contacts in industry and research establishments, I know that this frequency expression has been used extensively and thus the paper served its purpose. At first sight, this extensive use might be the reason for its frequent citation, but most users had practical problems to solve and were unlikely to publish a related paper. The dramatic increase in interest in plate vibrations in subsequent years, the simplicity of my approach, and the possibility of similar treatment of related problems contributed to the number of citations. Nevertheless, it surprises me that citations continued after the publication of Leissa's excellent monograph in 1969,[4] which surveyed all previously published work on plate vibrations and reduced the necessity to refer to earlier work.

"The paper might never have been written. At that time, I was a research fellow on secondment from my lectureship; my assignment was in the area that is now known as dynamic soil-structure interaction. Thus, this work on plates was a digression, which was undertaken in the best tradition of academic freedom, formed a particularly interesting and satisfying project of short-term duration, and led to a continuing interest in the subject."[5]

1. **Rayleigh J W S.** *The theory of sound.* London: Macmillan, 1894. 2 vols.
2. **Warburton G B.** Rayleigh's contributions to modern vibration analysis. *J. Sound Vib.* **88**:163-73, 1983.
3. **Leissa A W.** The free vibration of rectangular plates. *J. Sound Vib.* **31**:257-93, 1973. (Cited 55 times.)
4. ---------------. *Vibration of plates.* Washington: National Aeronautics and Space Administration, 1969. NASA SP-160. (Cited 325 times.)
5. **Warburton G B.** Response using the Rayleigh-Ritz method. *Earthquake Eng. Struc. Dynam.* 7:327-34, 1979.