This Week's Citation Classic

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This paper by and large is a tutorial paper. It introduces pertinent state-variable concepts to network theory. The emphasis is on the formulation of state equations for classical networks due to Bashkow[1] and Bryant.[2] Also included are extensions to nonlinear, time-varying, and active networks. [The *SCI®* indicates that this paper has been cited in over 105 publications since 1965.]

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"Electrical engineering education and research changed drastically during the 1950s. By the early 1960s, the well-developed theory and techniques for linear, time-invariant circuits and systems based on the frequency-domain approach gradually lost their preeminent position because of the prevalence of computers and the need to face many large-scale real-world problems in electronics, communication, and control which are inherently nonlinear. The time-domain approach based on the state-variable formulation evolved almost simultaneously in control systems and network theory from the research community. The editor of *Proceedings of the IEEE* saw a crucial need to popularize it to electrical engineers in general, and especially to circuit designers, and invited me to write such a paper. I asked my colleague and former student Ron Rohrer, who at that time was also writing a book with me, to join me on the project. The paper was intended to serve three purposes, namely: (1) to review the fundamental concepts of the state-space techniques; (2) to give a complete treatment of the state-variable formulation for linear, time-invariant RLC circuits; and (3) to point out possible extensions and generalizations to nonlinear, time-varying, and active circuits.

"A brief account of the history of the evolution of the state-variable approach to network analysis is perhaps of interest here. In the early 1950s, a group of fresh PhDs working at Bell Laboratories in New Jersey, including Ted Bashkow, Charlie Desoer, Bill Gross, and myself, conducted an after-hours self-study course. We ran into the book *Stability Theory of Differential Equations* by R. Bellman.[3] All of us learned the subject of differential equations from the classical approach and were intrigued by the generality of the first order vector differential equation. We began to wonder whether electric networks, which are historically analyzed by loop or node equations using Kirchhoff's laws and expressed in the form of integrodifferential equations, can be formulated in the form of first order vector differential equations, later called state equations. It was Bashkow who, with considerable persistence and hard work, finally succeeded in writing a special class of linear networks in the desired form.[1] Following Bellman, Bashkow called the resulting matrix, A-matrix. The method was later generalized to all linear, time-invariant RLC networks by Bryant using a graph theoretical approach.[2]

"Almost concurrently, control theorists were working on stability of nonlinear systems based on Lyapunov's second method.[4] Considerable amounts of conceptual and qualitative results on linear systems were developed using the vector differential equation.[5] The term state variable was introduced to connect physical entities to the mathematical formulation. Subsequently, major advances have been made in control systems, system theory, and nonlinear networks."

1. **Bashkow T R.** The A matrix, a new network description. *IRE Trans. Circuit Theory* **CT-4**:117-20, 1957.
2. **Bryant B R.** The explicit form of Bashkow's A matrix. *IRE Trans. Circuit Theory* **CT-9**:303-6, 1962.
3. **Bellman R.** *Stability theory of differential equations.* New York: McGraw-Hill, 1953. 166 p.
4. **Kalman R E & Bertram J E.** Control system analysis and design via the "second method" of Lyapunov. *Trans. ASME Ser. D—J. Basic Eng.* **82**:371-400, 1960.
5. **Zadeh L A & Desoer C A.** *Linear system theory: the state space approach.* New York: McGraw-Hill, 1963. 628 p.