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This Week's Citation Classic

Finlayson B A & Scriven L E. The method of weighted residuals—a review.
Appl. Mech. Rev. **19**:735-48, 1966.
[University of Minnesota, Minneapolis, MN]

The method of weighted residuals (MWR) is described and presented in its historical context. The method provides a unifying theme for several diverse methods for solving differential equations. The relationship of the Galerkin method[1] with variational methods is outlined. [The SCI® indicates that this paper has been explicitly cited in about 100 publications, making it the most-cited paper published in this journal.]

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"This publication originated during my graduate student days. Despite drawbacks like financial stringency, graduate study has benefits as well, like the possibility to focus on one issue and to read in the library at leisure (almost!). I was blessed with excellent library facilities at the University of Minnesota: open stacks and extensive holdings. The method of weighted residuals (MWR) had developed over a period of 50 years, and I needed to read the original papers; fortunately, the dusty volumes were there. If I had to resort to interlibrary loan, this paper would never have been published! While studying these dusty volumes, it was clear there was a unity in, and relationship between, the various methods used for approximating solutions to differential equations. The material was finally summarized as a chapter in my thesis.

"Careful comparison revealed that methods advanced by Biot[2] and Prigogine[3] were only disguised applications of the Galerkin method. This cleared the decks for the emergence of the finite element method[4] based on the Galerkin method, without the encumbrance of these so-called variational principles.

"As a graduate student I had viewed the work as interesting and fun to do, but its usefulness to others was not obvious to me. Scriven encouraged (insisted!) it be written for publication. I was leaving for a two-year stint in the Navy at the Office of Naval Research in Washington, DC. The Library of Congress was invaluable as a source for rechecking key references during manuscript preparation.

"The methods for solving differential equations that are discussed in the paper had been developed over a period of 50 years before exploding with the widespread use of the finite element method. The paper came at a critical time, since it summarized the known information at the end of an era, in preparation for the new direction. The paper provided a jumping-off point for someone to learn what Galerkin meant in the context of the finite element method. The subdomain method was also revealed as a forerunner of the finite element method.

"I knew the article had been cited often (my college asks us to count citations), but I didn't know the specific reasons each author used. Upon closer look, I found that some refer to it for information on MWR, others for the Galerkin method, while others use it to buttress their use or nonuse of a variational method. One reference even used it as a source of information about the orthogonal collocation method, even though that method had not yet been invented when our paper was published. The fields of citations are diverse: chemical reaction engineering, water waves, population balances, nuclear engineering, petroleum engineering, electrochemistry, and aerosols, to name a few. The broad application of the methods is another reason for the many citations.

"The paper influenced my career as well. It caught the eye of an editor who encouraged me to write a book on the method of weighted residuals,[5] which fleshes out the details only alluded to in the article. Probably another reason the article is cited is that it is much cheaper than the book! A consulting job that resulted from the cited article led to solution methods which have since been written in still another book.[6] The career impact of the cited article continues to this day."

1. **Galerkin B G.** Sterzhni i plastiny. Ryady v nekotorykh voprosakh uprogogo ravnovesiya sterzhnei i plastin. *Vestn. Inzhen. Tekh. Petrograd* **19**:897-908, 1915.
2. **Biot M A.** New methods in heat flow analysis with application to flight structures. *J. Aerosol Sci.* **24**:857-73, 1957. (Cited 80 times.)
3. **Glansdorff P, Prigogine I & Hays D F.** Variational properties of a viscous liquid at a nonuniform temperature. *Phys. Fluids* **5**:144-9, 1962.
4. **Zienkiewicz O C.** *The finite element method.* New York: McGraw-Hill, 1977. 787 p.
5. **Finlayson B A.** *The method of weighted residuals and variational principles, with application in fluid mechanics, heat and mass transfer.* New York: Academic Press, 1972. 412 p. (Cited 370 times.)
6. ----------. *Nonlinear analysis in chemical engineering.* New York: McGraw-Hill, 1980. 366 p.

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