

Callaway J. Model for lattice thermal conductivity at low temperatures. *Phys. Rev.* 113:1046-51, 1959.  
[Westinghouse Research Laboratories, Pittsburgh, PA]

The thermal conductivity of a semiconductor or insulator can be calculated at low temperatures with the use of a simple model in which one adds reciprocal relaxation times for phonon scattering processes. There is a correction which allows for the conservation of the total crystal momentum by normal three-phonon scattering. [The  $SC|^{\circ}$  indicates that this paper has been cited in over 490 publications since 1961.]

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"This work grew out of a summer I spent at the Westinghouse Research Laboratories in Pittsburgh in 1958. I was at that time a junior faculty member of the University of Miami, and was in the habit of escaping from the then not air-conditioned premises during the summer months to work on different topics in a pleasant climate. This was a period in which there was interest in the possible thermoelectric generation of electrical power. The materials of interest were, in the main, semiconductors. It turns out that one of the fundamental material parameters determining the possible efficiency of this process is the thermal conductivity, and I began to think about the theory of the thermal conductivity of semiconductors and insulators, in which the heat current is carried primarily by phonons (in contrast to metals, where free electrons transport thermal energy).

"The theory at that time was in a very unsatisfactory condition. For example, although localized (point) impurities, such as an atom of different

mass, could scatter phonons, it was not clear how one could obtain the contribution of a given concentration of impurities to the thermal resistance. The difficulty was that point defects are very ineffective in scattering long wavelength lattice waves (the cross section is proportional to  $\omega^4$  for small  $\omega$ ) so that point impurity scattering can not by itself lead to a finite thermal resistance. On the other hand, normal anharmonic three-phonon interactions scatter long wavelength phonons adequately but also by themselves do not lead to a finite thermal resistance because they conserve the total crystal momentum, and therefore the heat current. One also needs to take boundary scattering and three-phonon Umklapp processes (which do not conserve crystal momentum) into account.

"The trick is to find a way to combine different phonon scattering processes in a way which respects the momentum conserving nature of the normal three-phonon scattering. The paper was successful because it proposed a way to do this in a reasonable, although approximate, manner. The immediate result was that I could account for the difference between the thermal resistivities of normal (i.e., isotopically disordered) and single isotope germanium crystals over a wide range of temperatures. The procedures I developed have proved useful to many authors in interpreting numerous experimental measurements of thermal conductivity in semiconductors and insulators, and enable estimation of the effects of changing material parameters. The fundamental principles of this approach have been investigated and refined by several authors (I cite three here<sup>1-3</sup>). An example of a more modern investigation using these principles, and also relating to germanium, can be found in reference 4."

1. Nettleton R E. Foundations of the Callaway theory of thermal conductivity. *Phys. Rev.* 132:2032-8, 1963.
2. Krumbhals J A. Thermal conductivity of insulating crystals in the presence of normal processes. *Proc. Phys. Soc. London* 85:921-30, 1965.
3. Simons S. Formulation and use of a model for the phonon Umklapp collision operator. *J. Phys.—C—Solid State Phys.* 8:1147-58, 1975.
4. Srivastava G P. Calculation of lattice thermal conductivity of Ge from 4 to 900 K. *Phil. Mag.* 34:795-809, 1976.