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This Week's Citation Classic

Tarjan R. Depth-first search and linear graph algorithms.
SIAM J. Comput. **1**:146-60, 1972.
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The fundamental properties of depth-first search as a tool for building efficient graph algorithms are presented and illustrated by means of two linear-time algorithms, for finding the biconnected components of an undirected graph and for finding the strong components of a directed graph. [The ***SCI®*** **indicates that this paper has been cited in over 105 publications since 1972, making it the most-cited paper published in this journal to date.]**

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"In the summer of 1970, just after my first year as a graduate student at Stanford University, John Hopcroft arrived on sabbatical from Cornell University. There ensued a very fruitful collaboration. We began work almost immediately on efficient graph algorithms. John was interested in finding problems for which one could prove nonlinear lower bounds on computation time. I suggested the problem of computing the biconnected components of a graph as a good candidate for a problem taking nonlinear time, but John was able to devise a linear-time algorithm for this problem. The main technique used in the algorithm is depth-first search. Depth-first search was proposed long ago by Trémaux[1] as a maze-searching method, but its theoretical properties had not been fully exploited at the time we began our work. We were able to obtain linear-time algorithms, based on depth-first search, for a number of graph problems, including finding triconnected components[2] and testing graph planarity.[3] The algorithm for the latter problem became the main part of my thesis.[4]

"I wrote 'Depth-first search and linear graph algorithms' with two purposes: to explain the theoretical properties of depth-first search, and to illustrate how those properties could be used to devise efficient algorithms for problems on both undirected and directed graphs. As an example of the use of depth-first search on undirected graphs, I used a refined version of John's original biconnected components algorithm, which we also discussed in another paper.[5] As an example of the use of depth-first search on directed graphs, I devised an algorithm for finding the strongly connected components of a directed graph. The paper appeared originally in the proceedings of a conference.[6]

"I had some difficulty in getting the strong components algorithm to work correctly; my initial attempts had subtle bugs. Although the algorithm is simple to state and program and very efficient in practice, proving that it is correct is not easy (and is an interesting exercise for program verifiers, both human and mechanical), because the algorithm interleaves several calculations that must all proceed simultaneously for the method to work.

"I think the paper has been cited so often because the algorithms it presents are simple and widely useful, and because it lays out the properties of depth-first search on which all depth-first search algorithms depend. Our other depth-first search algorithms are both more complicated and more specialized than the ones presented in the paper, and thus are less widely cited. The substance of the paper appears in many algorithms texts, of which the seminal one is by Aho, Hopcroft, and Ullman."[7]

1. **Lucas E.** *Récréations mathématiques.* Paris: Gauthier-Villars, 1883. Vol. I.
2. **Hopcroft J E & Tarjan R E.** Dividing a graph into triconnected components. *SIAM J. Comput.* **2**:135-58, 1973.
3. ———————. Efficient planarity testing. *J. Assn. Comput. Mach.* **21**:549-68, 1974.
4. **Tarjan R E.** *An efficient planarity algorithm.* Stanford, CA: Stanford University, Computer Science Department, 1971. Technical Report STAN-CS-244-71.
5. **Hopcroft J E & Tarjan R E.** Algorithm 447: efficient algorithms for graph manipulation. *Commun. ACM* **16**:372-8, 1973.
6. **Tarjan R E.** Depth-first search and linear graph algorithms. *Conference record of Twelfth Annual IEEE Symposium on Switching and Automata Theory,* 13-15 October 1971. New York: IEEE, 1971. p. 114-21.
7. **Aho A V, Hopcroft J E & Ullman J D.** *The design and analysis of computer algorithms.* Reading, MA: Addison-Wesley, 1974. 470 p.