A theoretical discussion is given of ranges of
heavy ions passing through matter with
moderate velocities, based on a simple scal-
ing theory of ion-atom collisions. Probabili-
ty distributions in total range and various
averages are studied, as well as ranges pro-
jected on initial direction of motion. [The
SCI® indicates that this paper has been cited
in over 2,240 publications since 1963.]

Jens Lindhard
Institute of Physics
University of Aarhus
DK 8000 Aarhus C
Denmark

February 8, 1983

"The main background of this paper is
Niels Bohr's celebrated survey of atomic
collisions, conceived before World War II,
but published in 1948.[1] From Bohr's discus-
sion we learned to look for the simple
essence of physical phenomena; to value ex-
periments and their results, but to view them
critically; and, not least, to understand
when to apply classical mechanics, instead
of quantum theory. Soon after, in connec-
tion with measurements by N.O. Lassen, it
was realized that, for heavy ions carrying
electrons, the ion charge theory of elec-
tronic stopping was somewhat uncertain.

"In the mid-1950s, an energy loss
measurement by R.B. Leachman and H. At-
terling was found not to be in agreement
with Bohr's simplified version of ion-atom
scattering. Morten Scharff and I began to
remedy this, but we gradually conceived the
ambitious plan of forming a comprehensive
theory of collisions of all ions with all
atoms, at moderate and low ion velocities.
In fact, we were already experienced in
Thomas-Fermi scaling properties in pertur-
bation cases, as well as in dynamic proper-

ties of electron gases. At low velocities, ion
scattering by atoms is important, but seems
to be complicated. We were slow in discov-
ering that a universal formula holds approx-
imately for differential cross sections in ion
scattering. A slightly different scaling was
obtained in the average energy loss to elec-
trons.

"The foundations of the work were laid,
and a group of four papers conceived, with
the common subtitle: 'Notes on atomic col-
lisions.' They embraced the theory and its
consequences. We were able to finish a
brief summary before Scharff died, in 1961,
at the age of 34. A group of young scientists,
Vibeke Nielsen, Hans Schiøtt, and P.V.
Thomsen, joined the work, and by a sus-
tained effort, three papers[2,3] were published
in the years 1963-1968.

"The present paper does not give a deriva-
tion of the collision theory, being merely an
application of the theory. Its topic was a
study of ranges of heavy ions passing
through matter, and probability distribu-
tions of ranges. This required a detailed in-
vestigation of integro-differential equations
belonging to evolution of probabilities. The
comprehensive solutions were meant to
represent an average behaviour, disregard-
ing individual deviations due to quantal
shell resonances in ions or atoms.

"Our paper allowed quite direct compari-
sons between experiments and theory. It so
happened that many stopping measure-
ments were performed in the 1960s with ac-
celerators originally used in nuclear physics,
and also new, interesting phenomena were
revealed within the physics of atomic colli-
sions in solids. Moreover, direct applications
of ion implantation were found, for instance
in electronic devices. To this end, several
books have been published containing
tables of the simple range formulae.
Because of such coincidences, seemingly,
the paper has been quoted a number of
times, and more frequently than the other
two papers."

1. **Bohr N.** On the penetration of charged particles through matter.
    Mat. Fys. Medd. Dan. Vid. Selsk. 18(18):1-144, 1948.
2. **Lindhard J, Nielsen V, Scharff M & Thomsen P V.** Integral equations governing radiation effects. (Notes on atomic
    collisions, III.) *Mat. Fys. Medd. Dan. Vid. Selsk.* 33(10):1-42, 1963.
3. **Lindhard J, Nielsen V & Scharff M.** Approximation method in classical scattering by screened coulomb
    fields. (Notes on atomic collisions, I.) *Mat. Fys. Medd. Dan. Vid. Selsk.* 36:(10):1-32, 1968.