This Week's Citation Classic

A classical many-body system is approximated by the use of collective coordinates. The two-body correlation function is required as input, but reappears as output. A self-consistent integral equation is thereby obtained, which is solved in virial expansion and tested—with gratifying results—on a hard sphere fluid. [The *SCI®* indicates that this paper has been cited in over 550 publications since 1961.]

Jerome K. Percus
Courant Institute of Mathematical Sciences
and Department of Physics
New York University
New York, NY 10012

August 29, 1982

"Two threads finally entwined to produce this paper. First was the Mayer diagram analysis of classical statistical mechanics,[1] my entrée into the field. I had been teaching electrical engineering in 1952-1954 and had just published a paper on reduction of circuits. At least the series-parallel reduction could be applied to Mayer diagrams, and I derived what later became known as the CHNC approximation. I wasn't sure what this was good for—nor were the people I spoke to—and just made it into an ONR report in 1956. When Emi Meeron organized a rump session of the APS in 1958, I presented this, and Bob Brout, who was circuit-wise, asked at once if Y-Δ reduction could be used as well. The answer was no, as Emi knew since he had performed a similar analysis while moonlighting at Hughes Aircraft a couple of years prior. But it was clear that diagrammatic analysis was a very useful tool—I was about to present the dynamical version in a course at New York University, and—but see below.

"Now back to 1952-1954 for the second crucial thread. George Yevick was very well acquainted with the Bohm-Pines normal mode analysis[2] of quantum systems, and he and I tried to set up a simplified version for classical thermal equilibrium as well as for condensed Bose systems. We succeeded in 1955 in recasting the problem exactly in terms of dynamical frequencies, and used their averages self-consistently as an approximation. But this could be done in too many ways, and the primitive computers available did not enable us to make any incisive comparisons with real data. It was at this frustrating point that we concluded that if our analysis was to make physical sense, the existence of effective normal mode frequencies had to be believed. But precisely what were they? Now diagrams came to the rescue. A diagrammatic density expansion showed that at least to low order, the frequencies were closely related to the pair distribution function, and the PY approximation of this paper was born. We then rederived it from the sampled wave vector viewpoint, one of our earliest tools, and sequential corrections were obtained somewhat later,[3] mainly as the result of work with Joel Lebowitz.

"The extensive citation of this paper derives from a few sources. To start, it was mathematically simple, which endeared it to Jack Kirkwood, for one, at first exposure. Second, computers could solve it to fair accuracy, and Art Broyles, a many-body expert as well, played a pivotal role in comparing such computation with experimental results. Third, for the basic model of interacting hard cores, it was solved analytically by Mike Wertheim[4]—and independently by Everett Thiele—following the diagrammatic computation of George Stell which showed that to several orders in density, only a cubic radial polynomial was required for the direct correlation. Personally, I regard the paper as a 'classic' only in the sense that it represents the melding of a firm physical theme—that of normal modes—with a modern mathematical tool—that of graph analysis—which is certainly the classical mode of progress in theoretical physics. For a recent review of the field see *Theory of Simple Liquids*[5] or *Many-Particle Physics*."[6]

1. Mayer J E & Mayer M G. *Statistical mechanics.* New York: Wiley, 1940. 495 p.
2. Bohm D & Pines D. A collective description of electron interaction III. *Phys. Rev.* 92:609-25, 1953.
3. Percus J K. The pair distribution function in classical statistical mechanics. (Frisch H L & Lebowitz J L, eds.) *The equilibrium theory of classical fluids.* New York: Benjamin, 1964. p. II-33-II-170.
4. Wertheim M S. Exact solution of the Percus-Yevick integral equation for hard spheres. *Phys. Rev. Lett.* 10:321-3, 1963.
5. Hansen J P & McDonald I R. *Theory of simple liquids.* New York: Academic Press, 1976. 395 p.
6. Mahan G D. *Many-particle physics.* New York: Plenum Press, 1981. 1003 p.