This Week's Citation Classic

Murtagh B A & Sargent R W H. Computational experience with quadratically convergent minimisation methods. *Computer J.* 13:185-94, 1970.
[Imperial College, London, England]

A recently reported minimisation method allows great flexibility in choosing successive steps without losing the property of quadratic convergence, but special precautions are necessary to ensure ultimate convergence from an arbitrary point for general functions. The paper makes an analysis of the required conditions, which give rise to several possible algorithms, and results of these for a number of problems are presented and discussed. [The *SCI*® indicates that this paper has been cited in over 105 publications since 1970.]

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"In the late-1960s, there was considerable interest in variable-metric methods for optimization. The idea had been promoted by Davidon,[1] and Fletcher and Powell[2] (DFP) and offered a significant advance on the old gradient based hill-climbing methods. My interest was in using the variable-metric approach in the context of linearly and nonlinearly constrained optimization, and this was the subject of my PhD research at Imperial College, London, under the supervision of Roger Sargent. Donald Goldfarb at Princeton University had produced some particularly impressive results in extending the original DFP variable-metric formula to linearly constrained optimization using orthogonal projection operators.[3]

"My approach centred on devising a variable-metric projection operator, using an updating scheme which allowed steps of arbitrary size and direction; an idea which seemed important in view of the need to project a step onto a surface of active linear constraints and the possibility of encountering a new constraint. I had come up with a scheme for incorporating the constraint normal vectors within the variable-metric formula and had produced somewhat better results than Goldfarb. On the basis of this, Roger and I submitted a paper to a conference at the University of Keele in March 1968.[4] Just prior to typing the final draft, Roger came along with a symmetric rank-one formula which had all the properties we needed. After the initial excitement, we were both disappointed to discover that Charles Broyden[5] and Davidon[1] had independently devised the same formula (as also had a number of others as it turned out), but nevertheless it did produce even better results in the linearly constrained case than I had earlier produced, and these were included in the Keele paper.

"I personally think the Keele paper was more important than the above one, but being in a book, it did not receive nearly the same readership. The above paper is solely concerned with unconstrained optimization; although our interest stemmed from linearly constrained optimization, it turned out that the rank-one formula also performed well for unconstrained optimization. The paper was concerned with ways of overcoming some theoretical deficiencies in the formula which detracted from its ability to accommodate arbitrary steps (something which could not be fully exploited in unconstrained optimization anyway). Perhaps one reason why it has been cited frequently is that it contained, as well as a detailed theoretical analysis, a comprehensive set of results on computational experiments.

"Since that time, there has developed a large literature on variable-metric formulae, and many new ones have been devised and analyzed in depth. Both the original DFP formula and the rank-one formula have been largely superseded in favour of the more recently devised 'Complementary DFP' formula.[6] An excellent review of the formulae and a careful analysis of their convergence properties is contained in the paper by Dennis and Moré."[6]

1. **Davidon W C.** *Variable metric method for minimization.* Lemont, IL: Argonne National Laboratory, May 1959, AEC Research and Development Report. ANL-5990 (Rev.). 27 p.
2. **Fletcher R & Powell M J D.** A rapidly convergent descent method for minimization. *Computer J.* 6:163-8, 1963.
3. **Goldfarb D.** Extension of Davidon's variable metric method to maximization under linear inequality and equality constraints. *SIAM J. Appl. Math.* 17:739-64, 1969.
4. **Murtagh B A & Sargent R W H.** A constrained minimization method with quadratic convergence. (Fletcher R, ed.) *Optimization.* London: Academic Press, 1969. p. 215-46.
5. **Broyden C G.** Quasi-Newton methods and their application to function minimization. *Math. Comput.* 21:368-81, 1967.
6. **Dennis J E, Jr. & Moré J J.** Quasi-Newton methods, motivation and theory. *SIAM Rev.* 19:46-89, 1977.